Proof. The right part of the above formula μ is a graph of an almost sub-join space (lemma).

That μ is an upper bound of S is obvious.

It remains to prove that μ is the least upper bound.

Suppose ν is an upper bound of S. Then $\nu \supseteq \bigcup S$. Thus, because ν is an almost sub-join-semilattice, $Z_\infty(\nu) \subseteq \nu$, likewise $Z_\infty(Z_\infty(\nu)) \subseteq \nu$, etc. Consequently $Z_\infty(\bigcup S) \subseteq \nu$, $Z_\infty(Z_\infty(\bigcup S)) \subseteq \nu$, etc. So we have $\mu \sqsubseteq \nu$. □

Conjecture 2342.

1°. $$\bigsqcup^{\mathbf{CASJ}} S = \left\{ \frac{\bigsqcup T_0 \sqcup \cdots \sqcup \bigsqcup T_{n-1}}{\begin{array}{l} n \in \mathbb{N}, T_0, \dots, T_{n-1} \in \bigcup S, \\ \bigsqcap T_0 \neq \bot \wedge \cdots \wedge \bigsqcap T_{n-1} \neq \bot, \\ \bigsqcup T_0 \not\asymp \bigsqcup T_1 \wedge \cdots \wedge \bigsqcup T_{n-2} \not\asymp \bigsqcup T_{n-1}. \end{array}} \right\};$$

2°. $$\bigsqcup^{\mathbf{CASJ}} S = \left\{ \frac{\bigsqcup T_0 \sqcup \bigsqcup T_1 \sqcup \dots}{\begin{array}{l} T_0, T_1, \dots \in \bigcup S, \\ \bigsqcap T_0 \neq \bot \wedge \bigsqcap T_2 \neq \bot \wedge \dots, \bigsqcup T_0 \not\asymp \bigsqcup T_1 \wedge \bigsqcup T_1 \not\asymp \bigsqcup T_2 \wedge \dots \end{array}} \right\}.$$

7. Up-complete low spaces

Definition 2343. *Ideal base* is a nonempty subset S of a poset such that $\forall a, b \in S \exists c \in S : (a, b \sqsubseteq c)$.

Obvious 2344. Ideal base is dual of filter base.

Theorem 2345. Product of nonempty posets is a ideal base iff every factor is an ideal base.

Proof. FiXme: more detailed proof

In one direction it is easy: Suppose one multiplier is not a dcpo. Take a chain with fixed elements (thanks our posets are nonempty) from other multipliers and for this multiplier take the values which form a chain without the join. This proves that the product is not a dcpo.

Let now every factor is dcpo. S is a filter base in $\prod \mathfrak{A}$ iff each component is a filter base. Each component has a join. Thus by proposition 638 S has a componentwise join. □

Definition 2346. I call a low space *up-complete* when each ideal base (or equivalently every nonempty chain, see theorem 586) in this space has join in this space.

Remark 2347. Elements of this ideal base are filters. (Thus is could be called a generalized ideal base.)

Example 2348.

1°. $\left\{ \frac{\mathcal{X} \in \mathfrak{F}]0;+\infty[}{\exists \varepsilon > 0 : \mathcal{X} \sqsubseteq \uparrow]\varepsilon;+\infty[} \right\} \cup \uparrow \{0\}$ is a graph of Cauchy space on $\mathbb{R}_+$, but not up-complete.

2°. $\mathfrak{F}[0;+\infty[$ is a strictly greater graph of Cauchy space on $\mathbb{R}_+$ and is up-complete.