PROOF. Let $\mathcal{F}$ be a proper Cauchy filter. Then $\bigsqcup^{\left\{\frac{\mathcal{X}\in\mathscr{C}}{\mathcal{X}\sqsupseteq\mathcal{F}}\right\}}\left\{\frac{\mathcal{X}\in\mathscr{C}}{\mathcal{X}\sqsupseteq\mathcal{F}}\right\}$ (existing by the above proposition) is the maximal Cauchy filter containing $\mathcal{F}$.

Suppose another maximal Cauchy filter $\mathcal{T}$ contains $\mathcal{F}$. Then $\mathcal{T}\in\left\{\frac{\mathcal{X}\in\mathscr{C}}{\mathcal{X}\sqsupseteq\mathcal{F}}\right\}$ and thus $\mathcal{T}=\bigsqcup^{\left\{\frac{\mathcal{X}\in\mathscr{C}}{\mathcal{X}\sqsupseteq\mathcal{F}}\right\}}\left\{\frac{\mathcal{X}\in\mathscr{C}}{\mathcal{X}\sqsupseteq\mathcal{F}}\right\}$. □

5. Relationships with symmetric reloids

FiXme: Also consider relationships with funcoids.

DEFINITION 2311. Denote $(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})=\bigsqcup\left\{\frac{\mathcal{X}\times^{\mathsf{RLD}}\mathcal{X}}{\mathcal{X}\in\mathscr{C}}\right\}$.

DEFINITION 2312. $(\mathrm{Low})\nu$ (*low space* for endoreloid ν) is a low space on U such that

$$\mathrm{GR}(\mathrm{Low})\nu=\left\{\frac{\mathcal{X}\in\mathscr{F}(U)}{\mathcal{X}\times^{\mathsf{RLD}}\mathcal{X}\sqsubseteq\nu}\right\}.$$

THEOREM 2313. If $(U,\mathscr{C})$ is a low space, then $(U,\mathscr{C})=(\mathrm{Low})(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$.

PROOF. If $\mathcal{X}\in\mathscr{C}$ then $\mathcal{X}\times^{\mathsf{RLD}}\mathcal{X}\sqsubseteq(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$ and thus $\mathcal{X}\in\mathrm{GR}(\mathrm{Low})(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$. Thus $(U,\mathscr{C})\sqsubseteq(\mathrm{Low})(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$.

Let's prove $(U,\mathscr{C})\sqsupseteq(\mathrm{Low})(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$.

Let $\mathcal{A}\in\mathrm{GR}(\mathrm{Low})(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$. We need to prove $\mathcal{A}\in\mathscr{C}$.

Really $\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A}\sqsubseteq(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$. It is enough to prove that $\exists\mathcal{X}\in\mathscr{C}:\mathcal{A}\sqsubseteq\mathcal{X}$.

Suppose $\nexists\mathcal{X}\in\mathscr{C}:\mathcal{A}\sqsubseteq\mathcal{X}$.

For every $\mathcal{X}\in\mathscr{C}$ obtain $X_{\mathcal{X}}\in\mathcal{X}$ such that $X_{\mathcal{X}}\notin\mathcal{A}$ (if forall $X\in\mathcal{X}$ we have $X_{\mathcal{X}}\in\mathcal{A}$, then $\mathcal{X}\sqsupseteq\mathcal{A}$ what is contrary to our supposition).

It is now enough to prove $\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A}\not\sqsubseteq\bigsqcup\left\{\frac{\uparrow^U X_{\mathcal{X}}\times^{\mathsf{RLD}}\uparrow^U X_{\mathcal{X}}}{\mathcal{X}\in\mathscr{C}}\right\}$.

Really, $\bigsqcup\left\{\frac{\uparrow^U X_{\mathcal{X}}\times^{\mathsf{RLD}}\uparrow^U X_{\mathcal{X}}}{\mathcal{X}\in\mathscr{C}}\right\}=\uparrow^{\mathsf{RLD}(U,U)}\bigcup\left\{\frac{\uparrow^U X_{\mathcal{X}}\times^{\mathsf{RLD}}\uparrow^U X_{\mathcal{X}}}{\mathcal{X}\in\mathscr{C}}\right\}$. So our claim takes the form $\bigcup\left\{\frac{\uparrow^U X_{\mathcal{X}}\times^{\mathsf{RLD}}\uparrow^U X_{\mathcal{X}}}{\mathcal{X}\in\mathscr{C}}\right\}\notin\mathrm{GR}(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{A})$ that is $\forall A\in\mathcal{A}:\bigcup\left\{\frac{\uparrow^U X_{\mathcal{X}}\times^{\mathsf{RLD}}\uparrow^U X_{\mathcal{X}}}{\mathcal{X}\in\mathscr{C}}\right\}\not\supseteq A\times A$ what is true because $X_{\mathcal{X}}\not\supseteq A$ for every $A\in\mathcal{A}$. □

REMARK 2314. The last theorem does not hold with $\mathcal{X}\times^{\mathsf{FCD}}\mathcal{X}$ instead of $\mathcal{X}\times^{\mathsf{RLD}}\mathcal{X}$ (take $\mathscr{C}=\left\{\frac{\{x\}}{x\in U}\right\}$ for an infinite set U as a counter-example).

REMARK 2315. Not every symmetric reloid is in the form $(\mathsf{RLD})_{\mathrm{Low}}(U,\mathscr{C})$ for some Cauchy space $(U,\mathscr{C})$. The same Cauchy space can be induced by different uniform spaces. See http://math.stackexchange.com/questions/702182/different-uniform-spaces-having-the-same-set-of-cauchy-filters

PROPOSITION 2316.

1°. $(\mathrm{Low})f$ is reflexive iff endoreloid f is reflexive.
2°. $(\mathsf{RLD})_{\mathrm{Low}}f$ is reflexive iff low space f is reflexive.

PROOF.

1°. f is reflexive $\Leftrightarrow 1^{\mathsf{RLD}}\sqsubseteq f\Leftrightarrow\forall x\in\mathrm{Ob}\,f:\uparrow(\{x\}\times\{x\})\sqsubseteq f\Leftrightarrow\forall x\in\mathrm{Ob}\,f:\uparrow\{x\}\times^{\mathsf{RLD}}\uparrow\{x\}\sqsubseteq f\Leftrightarrow\forall x\in\mathrm{Ob}\,f:\uparrow\{x\}\in(\mathrm{Low})f\Leftrightarrow(\mathrm{Low})f$ is reflexive.

2°. Let f is reflexive. Then $\forall x\in\mathrm{Ob}\,f:\uparrow\{x\}\in f$; $\forall x\in\mathrm{Ob}\,f:\uparrow\{x\}\times^{\mathsf{RLD}}\uparrow\{x\}\sqsubseteq(\mathsf{RLD})_{\mathrm{Low}}f$; $\forall x\in\mathrm{Ob}\,f:\uparrow(\{x\}\times\{x\})\sqsubseteq(\mathsf{RLD})_{\mathrm{Low}}f$; $1^{\mathsf{RLD}}\sqsubseteq(\mathsf{RLD})_{\mathrm{Low}}f$.

Let now $(\mathsf{RLD})_{\mathrm{Low}}f$ be reflexive. Then $f=(\mathrm{Low})(\mathsf{RLD})_{\mathrm{Low}}f$ is reflexive. □