CHAPTER 11

Cauchy Filters on Reloids

In this chapter I consider *low filters* on reloids, generalizing Cauchy filters on uniform spaces. Using low filters, I define Cauchy-complete reloids, generalizing complete uniform spaces.

FiXme: I forgot to note that Cauchy spaces induce topological (or convergence) spaces.

1. Preface

Replace `\langle ...\rangle` with `\supfun{...}` in LaTeX.

This is a preliminary partial draft.

To understand this article you need first look into my book [2].

http://math.stackexchange.com/questions/401989/what-are-interesting-properties-of-totally-bounded-uniform-spaces

http://ncatlab.org/nlab/show/proximity+space#uniform_spaces for a proof sketch that proximities correspond to totally bounded uniformities.

2. Low spaces

FiXme: Analyze http://link.springer.com/article/10.1007/s10474-011-0136-9 ("A note on Cauchy spaces"), http://link.springer.com/article/10.1007/BF00873992 ("Filter spaces"). It also contains references to some useful results, including ("On continuity structures and spaces of mappings" freely available at https://eudml.org/doc/16128) that the category FIL of filter spaces is isomorphic to the category of filter merotopic spaces (copy its definition).

DEFINITION 2290. A *lower set*[1] of filters on U (a set) is a set $\mathscr{C}$ of filters on U, such that if $\mathcal{G} \sqsubseteq \mathcal{F}$ and $\mathcal{F} \in \mathscr{C}$ then $\mathcal{G} \in \mathscr{C}$.

REMARK 2291. Note that we are particularly interested in nonempty (= containing the improper filter) lower sets of filters. This does not match the traditional theory of Cauchy spaces (see below) which are traditionally defined as not containing empty set. Allowing them to contain empty set has some advantages:

- Meet of any lower filters is a lower filter.
- Some formulas become a little simpler.

DEFINITION 2292. I call *low space* a set together with a nonempty lower set of filters on this set. Elements of a (given) low space are called *Cauchy filters*.

DEFINITION 2293. $\operatorname{GR}(U, \mathscr{C}) = \mathscr{C}$; $\operatorname{Ob}(U, \mathscr{C}) = U$. $\operatorname{GR}(U, \mathscr{C})$ is read as *graph* of space $(U, \mathscr{C})$. I denote $\operatorname{Low}(U)$ the set of graphs of low spaces on the set U. Similarly I will denote its subsets $\mathbf{ASJ}(U)$, $\mathbf{CASJ}(U)$, $\mathbf{Cau}(U)$, $\mathbf{CCau}(U)$ (see below).

FiXme: Should use "space structure" instead of "graph of space", to match customary terminology.

[1] Remember that our orders on filters is the reverse to set theoretic inclusion. It could be called an *upper* set in other sources.