CHAPTER 10

Quasi-atoms

DEFINITION 2285. *Quasi-atoms* funcoid $\mathscr{A}$ is the funcoid $A \to \operatorname{atoms}^{\mathfrak{A}} A$ defined by the formula $\langle \mathscr{A} \rangle^* X = \operatorname{atoms}^{\mathfrak{A}} X$.

This really defines a funcoid because $\operatorname{atoms}^{\mathfrak{A}} \bot = \emptyset$ and $\operatorname{atoms}^{\mathfrak{A}}(X \cup Y) = \operatorname{atoms}^{\mathfrak{A}} X \cup \operatorname{atoms}^{\mathfrak{A}} Y$.

OBVIOUS 2286. $\mathscr{A}$ is a co-complete funcoid.

PROPOSITION 2287. $\left\langle \mathscr{A}^{-1} \right\rangle^* Y = \bigsqcup Y$.

PROOF. $Y \not\asymp \langle \mathscr{A} \rangle^* X \Leftrightarrow Y \not\asymp \operatorname{atoms}^{\mathfrak{A}} X \Leftrightarrow \exists x \in \operatorname{atoms}^{\mathfrak{A}} X, y \in Y : x \not\asymp y \Leftrightarrow \exists y \in Y : X \not\asymp y \Leftrightarrow$ (because X is a principal filter) $\Leftrightarrow X \not\asymp \bigsqcup Y$. □

Note $\langle \mathscr{A} \rangle^* \mathcal{X} = \bigsqcap^{\mathscr{F}}_{X \in \operatorname{up} \mathcal{X}} \operatorname{atoms}^{\mathfrak{A}} X$;

$\left\langle \mathscr{A}^{-1} \right\rangle^* \mathcal{Y} = \bigsqcap^{\mathscr{F}}_{Y \in \operatorname{up} \mathcal{Y}} \bigsqcup Y$ ($\mathcal{Y}$ is filter on the set of ultrafilters).

Can $\operatorname{atoms}^{\mathfrak{A}} \mathcal{X}$ be restored knowing $\langle \mathscr{A} \rangle \mathcal{X}$? Can $\bigsqcup \mathcal{Y}$ be restored knowing $\left\langle \mathscr{A}^{-1} \right\rangle \mathcal{X}$?

PROPOSITION 2288. (Provided that A is infinite) $\mathscr{A}$ is not complete.

PROOF. Take a nonprincipal ultrafilter x. Then $\left\langle \mathscr{A}^{-1} \right\rangle^* \{x\} = \bigsqcup \{x\} = x$ is a nonprincipal filter. □

CONJECTURE 2289. There is such filter $\mathcal{X}$ that $\langle \mathscr{A} \rangle^* \mathcal{X}$ is non-principal.

Does quasi-atoms funcoid define a more elegant replacement of $\operatorname{atoms}^{\mathfrak{A}}$? Does this concept have any use?