CHAPTER 9

Backward Funcoids

This is a preliminary partial draft.

Fix a family $\mathfrak{A}$ of posets.

Definition 2279. Let f be a staroid of filters $\mathfrak{F}(\mathfrak{A}_i)$ on boolean lattices $\mathfrak{A}_i$. *Backward funcoid* for the argument $k \in \operatorname{dom}\mathfrak{A}$ of f is the funcoid $\operatorname{Back}(f,k)$ defined by the formula (for every $X \in \mathfrak{A}_k$)

$$\langle \operatorname{Back}(f,k)\rangle X = \left\{ \frac{L \in \prod_{i \in \operatorname{dom}\mathfrak{A}} \mathfrak{F}(\mathfrak{A}_i)}{X \in \langle f \rangle_k L} \right\}.$$

Proposition 2280. Backward funcoid is properly defined.

Proof. $\langle \operatorname{Back}(f,k)\rangle^*(X \sqcup Y) = \left\{ \frac{L \in \prod \mathfrak{A}}{X \sqcup Y \in \langle f \rangle_k L} \right\} = \left\{ \frac{L \in \prod \mathfrak{A}}{X \in \langle f \rangle_k L \vee Y \in \langle f \rangle_k L} \right\} = \left\{ \frac{L \in \prod \mathfrak{A}}{X \in \langle f \rangle_k L} \right\} \cup \left\{ \frac{L \in \prod \mathfrak{A}}{Y \in \langle f \rangle_k L} \right\} = \langle \operatorname{Back}(f,k)\rangle^* X \cup \langle \operatorname{Back}(f,k)\rangle^* Y$. □

Obvious 2281. Backward funcoid is co-complete.

Proposition 2282. If f is a principal staroid then $\operatorname{Back}(f,k)$ is a complete funcoid.

Proof. ?? □

Proposition 2283. f can be restored from $\operatorname{Back}(f,k)$ (for every fixed k).

Proof. ?? □

Proposition 2284. $f \mapsto \operatorname{Back}(f,k)$ is an order isomorphism $\mathsf{Strd}^{\mathfrak{A}} \to \mathsf{FCD}\left(\mathfrak{A}_k, \mathsf{Strd}^{(\operatorname{dom}\mathfrak{A})\setminus\{k\}}\right)$.

Proof. ?? □