5. Negative results

The following negative result generalizes theorem 3.8 in [3].

THEOREM 2274. The element $1^{(\mathrm{Src}\,\Upsilon)(\mathfrak{A},\mathfrak{A})}$ is not complemented if $\mathfrak{A}$ is a non-atomic boolean lattice, for every monotone system of sides.

PROOF. Let $T = 1^{(\mathrm{Src}\,\Upsilon)(\mathfrak{A},\mathfrak{A})}$.

Let's suppose $T \sqcup V = \top$ for $V \in (\mathrm{Src}\,\Upsilon)(\mathfrak{A},\mathfrak{A})$ and prove $T \sqcap V \neq \bot$.

Then $\langle T \sqcup V\rangle a = \top$ for all $a \neq \bot$ and thus $\langle V\rangle a \sqcup a = \top$.

Consequently $\langle V\rangle a \sqsupseteq \neg a$ for all $a \neq \bot$.

If a isn't an atom, then there exists b with $0 \sqsubset b \sqsubset a$ and hence $\langle V\rangle a \sqsupseteq \langle V\rangle b \sqsupseteq \neg b \sqsupset \neg a$; thus $\langle V\rangle a \sqsupset \neg a$.

There is such $c \sqsubset \top$ that $a \sqsubseteq c$ for every atom a. (Really, suppose some element $p \neq \bot$ has no atoms. Thus all atoms are in $\neg p$.)

For $a \not\sqsubseteq c$ we have $\langle V\rangle a \sqcap a \sqsupset \bot$ for all $a \sqsubseteq \neg c$ thus $\langle T \sqcap V\rangle a \sqsupseteq \langle V\rangle a \sqcap a \sqsupset \bot$. Thus $\langle (T \sqcap V) \circ \mathrm{id}_{\neg c}\rangle a \sqsupset \bot$

So $T \sqcap V \sqsupseteq (T \sqcap V) \circ \mathrm{id}_{\neg c} \sqsupset \bot$. So V is not a complement of T. □

COROLLARY 2275. $(\mathrm{Src}\,\Upsilon)(\mathfrak{A},\mathfrak{A})$ is not boolean if $\mathfrak{A}$ is a non-atomic boolean lattice.

6. Dagger systems of sides

PROPOSITION 2276.

1°. For a partially ordered dagger category, each of Hom-set of which has least element, we have $\bot^\dagger = \bot$.

2°. For a partially ordered dagger category, each of Hom-set of which has greatest element, we have $\top^\dagger = \top$.

PROOF. $\forall f \in \mathrm{Hom}(A,B) : \bot^\dagger \sqsubseteq f \Leftrightarrow \forall f \in \mathrm{Hom}(A,B) : \bot \sqsubseteq f^\dagger \Leftrightarrow \forall f \in \mathrm{Hom}(A,B) : \bot \sqsubseteq f \Leftrightarrow 1$. Thus $\bot^\dagger$ is the least.

The other items is dual. □

DEFINITION 2277. *Dagger system of presides with identities* is system of presides with identities with category $\mathrm{Src}\,\Upsilon$ being a partially ordered dagger category and $(\mathrm{id}_X)^\dagger = \mathrm{id}_X$ for every X.

PROPOSITION 2278. For a system of sides we have $(X \times Y)^\dagger = Y \times X$.

PROOF. $(X \times Y)^\dagger = (\mathrm{id}_Y \circ \top \circ \mathrm{id}_X)^\dagger = \mathrm{id}_X^\dagger \circ \top^\dagger \circ \mathrm{id}_Y^\dagger = \mathrm{id}_X \circ \top \circ \mathrm{id}_Y = Y \times X$. □

FiXme: Which properties of pointfree funcoids can be generalized for sides?