

thus it is a system of presides.

That this is a bounded system of presides follows from the formulas $(\text{FCD})_{\perp}^{\text{RLD}(A,B)} = \perp$ and $(\text{FCD})_{\top}^{\text{RLD}(A,B)} = \top$.

It is with identities, because proposition 1168. It is with correct identities by proposition 898. \square

FiXme: Also for pointfree reloids.

FiXme: These examples works for (dagger) systems of sides with binary product.

4. Product

DEFINITION 2268. *Binary product* of objects of presides with identities is defined by the formula $X \times Y = \text{id}_Y \circ \top \circ \text{id}_X$.

DEFINITION 2269. System of presides with identities is *with correct binary product* when $f \sqcap (X \times Y) = \text{id}_Y \circ f \circ \text{id}_X$ for every preside f .

PROPOSITION 2270. $\langle A \times B \rangle X = \begin{cases} \perp & \text{if } X \simeq A \\ B & \text{if } X \not\simeq A \end{cases}$

PROOF.

$$\begin{aligned} \langle A \times B \rangle X &= \langle \text{id}_B \circ \top \circ \text{id}_A \rangle X = \langle \text{id}_B \rangle \langle \top \rangle \langle \text{id}_A \rangle X = \\ &= B \sqcap \langle \top \rangle (X \sqcap A) = B \sqcap \begin{cases} \perp & \text{if } X \simeq A \\ \top & \text{if } X \not\simeq A \end{cases} = \begin{cases} \perp & \text{if } X \simeq A \\ B & \text{if } X \not\simeq A \end{cases} \end{aligned}$$

\square

DEFINITION 2271. I will call a system of sides *with correct meet* when

$$(X_0 \times Y_0) \sqcap (X_1 \times Y_1) = (X_0 \sqcap X_1) \times (Y_0 \sqcap Y_1).$$

PROPOSITION 2272. Faithful systems of presides with identities are with correct meet.

PROOF. $(X_0 \times Y_0) \sqcap (X_1 \times Y_1) = \text{id}_{Y_1} \circ (X_0 \times Y_0) \circ \text{id}_{X_1}$. Thus

$$\begin{aligned} \langle (X_0 \times Y_0) \sqcap (X_1 \times Y_1) \rangle P &= \langle \text{id}_{Y_1} \rangle \langle X_0 \times Y_0 \rangle \langle \text{id}_{X_1} \rangle P = \\ &= \langle \text{id}_{Y_1} \rangle \begin{cases} \perp & \text{if } X_0 \simeq \langle \text{id}_{X_1} \rangle P \\ Y_0 & \text{if } X_0 \not\simeq \langle \text{id}_{X_1} \rangle P \end{cases} = \begin{cases} \perp & \text{if } X_0 \sqcap X_1 \simeq P \\ Y_0 \sqcap Y_1 & \text{if } X_0 \sqcap X_1 \not\simeq P \end{cases} = \\ &= \langle (X_0 \sqcap X_1) \times (Y_0 \sqcap Y_1) \rangle P. \end{aligned}$$

So $(X_0 \times Y_0) \sqcap (X_1 \times Y_1) = (X_0 \sqcap X_1) \times (Y_0 \sqcap Y_1)$ follows by full faithfulness. \square

PROPOSITION 2273. Systems of presides with correct identities are with correct meet.

PROOF. $(X_0 \times Y_0) \sqcap (X_1 \times Y_1) = \text{id}_{Y_1} \circ (X_0 \times Y_0) \circ \text{id}_{X_1} = \text{id}_{Y_1} \circ (\text{id}_{Y_0} \circ \top \circ \text{id}_{X_0}) \circ \text{id}_{X_1} = \text{id}_{Y_0 \sqcap Y_1} \circ \top \circ \text{id}_{X_0 \sqcap X_1} = (X_0 \sqcap X_1) \times (Y_0 \sqcap Y_1)$. \square

For some sides holds the formula $f \circ (X \times Y) = X \times \langle f \rangle Y$. I refrain to give a name for this property.