COROLLARY 2231. $\langle\bot\rangle x = \bot$ for Galois connections from a poset $\mathfrak{A}$ with greatest element to a poset $\mathfrak{B}$ with least element. FiXme: Clarify.

THEOREM 2232. If $\mathfrak{A}$ and $\mathfrak{B}$ are bounded posets, then $\mathsf{GAL}(\mathfrak{A},\mathfrak{B})$ is bounded.

PROOF. That $\mathsf{GAL}(\mathfrak{A},\mathfrak{B})$ has least element was proved above. I will demonstrate that (α,β) is the greatest element of $\mathsf{pFCD}(\mathfrak{A},\mathfrak{B})$ for

$$\alpha X = \begin{cases} \bot^{\mathfrak{B}} & \text{if } X = \bot^{\mathfrak{A}} \\ \top^{\mathfrak{B}} & \text{if } X \neq \bot^{\mathfrak{A}} \end{cases}; \quad \beta Y = \begin{cases} \top^{\mathfrak{A}} & \text{if } Y = \top^{\mathfrak{B}} \\ \bot^{\mathfrak{A}} & \text{if } Y \neq \top^{\mathfrak{B}} \end{cases}.$$

First prove $Y \sqsubseteq \alpha X \Leftrightarrow X \sqsubseteq \beta Y$.

Really $\alpha X \sqsubseteq Y \Leftrightarrow X = \bot^{\mathfrak{A}} \vee Y = \top^{\mathfrak{B}} \Leftrightarrow X \sqsubseteq \beta Y$.

That it is the greatest Galois connection between $\mathfrak{A}$ and $\mathfrak{B}$ easily follows from proposition 2229. □

THEOREM 2233. For every brouwerian lattice $x \mapsto c \sqcap x$ is a lower adjoint.

PROOF. By dual of theorem 154. □

EXERCISE 2234. Describe the corresponding upper adjoint, especially for the special case of boolean lattices.

2. Definition

DEFINITION 2235. *System of presides* is a functor $\Upsilon = (f \mapsto \langle f\rangle)$ from an ordered category to the category of functions between (small) bounded lattices, such that (for all relevant variables):

1°. Every Hom-set of $\operatorname{Src}\Upsilon$ is a bounded join-semilattice.
2°. $\langle a\rangle\bot = \bot$.
3°. $\langle a \sqcup b\rangle X = \langle a\rangle X \sqcup \langle b\rangle X$ (equivalent to Υ to be a join-semilattice homomorphism, if we order functions between small bounded lattices componentwise).

I call morphisms of such categories *sides*.[1]

REMARK 2236. We could generalize to functions between small join-semilattices with least elements instead of bounded lattices only, but this is not really necessary.

DEFINITION 2237. I will call objects of the source category of this functor simply *objects of the presides*.

DEFINITION 2238. *Bounded* system of presides is system of presides from an ordered category with bounded Hom-sets such that $X, Y \in \operatorname{Ob}\operatorname{Src}\Upsilon$ the following additional axioms hold for all suitable a:

1°. $\left\langle \bot^{\operatorname{Hom}(X,Y)} \right\rangle a = \bot$.
2°. $\left\langle \top^{\operatorname{Hom}(X,Y)} \right\rangle a = \top$ unless $a = \bot$

DEFINITION 2239. *System of presides with identities* is a system of presides with a morphism $\mathrm{id}_a \in \operatorname{Src}\Upsilon$ for every object $\mathfrak{A}$ of $\operatorname{Src}\Upsilon$ and $a \in \mathfrak{A}$ and the following additional axioms:

1°. $\mathrm{id}_c \sqsubseteq 1_{\mathfrak{A}}$ for every $c \in \mathfrak{A}$ where $\mathfrak{A}$ is an object of $\operatorname{Src}\Upsilon$.
2°. $\langle \mathrm{id}_c \rangle = (\lambda x \in \mathfrak{A} : x \sqcap c)$ for every $c \in \mathfrak{A}$ where $\mathfrak{A}$ is an object of $\operatorname{Src}\Upsilon$

DEFINITION 2240. *System of sides* is a system of presides which is both bounded and with identities.

[1]The idea for the name is that we consider one "side" $\langle f\rangle$ of a funcoid instead of both sides $\langle f\rangle$ and $\left\langle f^{-1}\right\rangle$.