CONJECTURE 2183. A d-path a is determined by the funcoids (where x spans $[0;1]$)

$$(\lambda t \in \mathbb{R} : a(x+t))|_{\Delta(0)}.$$

5. A way to construct directed topological spaces

I propose a new way to construct a directed topological space. My way is more geometric/topological as it does not involve dealing with particular paths.

CONJECTURE 2184. Every directed topological space can be constructed in the below described way.

Consider topological space T and its subfuncoid F (that is F is a funcoid which is less that T in the order of funcoids). Note that in our consideration F is an endofuncoid (its source and destination are the same).

Then a directed path from point A to point B is defined as a continuous function f from $[0;1]$ to F such that $f(0) = A$ and $f(1) = B$. FiXme: Specify whether the interval $[0;1]$ is treated as a proximity, pretopology, or preclosure.

Because F is less that T, we have that every directed path is a path.

CONJECTURE 2185. The two directed topological spaces, constructed from a fixed topological space and two different funcoids, are different.

For a counter-example of (which of the two?) the conjecture consider funcoid $T \sqcap (\mathbb{Q} \times^{\mathsf{FCD}} \mathbb{Q})$ where T is the usual topology on real line.We need to consider stability of existence and uniqueness of a path under transformations of our funcoid and under transformations of the vector field. Can this be a step to solve Navier-Stokes existence and smoothness problems?

6. Integral curves

We will consider paths in a normed vector space V.

DEFINITION 2186. Let D be a connected subset of $\mathbb{R}$. A *path* is a function $D \to V$.

Let d be a vector field in a normed vector space V.

DEFINITION 2187. *Integral curve* of a vector field d is a differentiable function $f : D \to V$ such that $f'(t) = d(f(t))$ for every $t \in D$.

DEFINITION 2188. The definition of *right side integral curve* is the above definition with right derivative of f instead of derivative f'. *Left side integral curve* is defined similarly.

6.1. Path reparameterization.

C^1 is a function which has continuous derivative on every point of the domain.

By D^1 I will denote a C^1 function whose derivative is either nonzero at every point or is zero everywhere.

DEFINITION 2189. A *reparameterization* of a C^1 path is a bijective C^1 function $\phi : D \to D$ such that $\phi'(t) > 0$. A curve f_2 is called a reparametrized curve f_1 if there is a reparameterization ϕ such that $f_2 = f_1 \circ \phi$.

It is well known that this defines an equivalence relation of functions.

PROPOSITION 2190. Reparameterization of D^1 function is D^1.

PROOF. If the function has zero derivative, it is obvious.

Let f_1 has everywhere nonzero derivative. Then $f_2'(t) = f_1'(\phi(t))\phi'(t)$ what is trivially nonzero. □