1°. $\mathscr{S}$ from a Hom-set $\mathsf{RLD}(A, B)$ to $\mathrm{End}_{\mathsf{RLD}}(\text{small sets})$ is an order embedding.
2°. $\mathscr{S}$ from the category RLD to $\mathrm{End}_{\mathsf{RLD}}(\text{small sets})$ is a prefunctor.
3°. $\mathscr{S}$ from unfixed reloids is an order embedding and a prefunctor (= semigroup homomorphism).

PROOF.

1°. That it's monotone is obvious. That it is an injection follows from $\mathscr{S}$ for filters being an injection.

2°. Let f and g be composable reloids.

If $H \in \operatorname{up} \mathscr{S}(g \circ f)$ then $H \supseteq H' \in \operatorname{up}(g \circ f)$, $H' \supseteq G \circ F$ for some H', $F \in \operatorname{up} f$ and $G \in \operatorname{up} g$. Consequently $F \in \operatorname{GR} \mathscr{S} f$, $G \in \operatorname{GR} \mathscr{S} g$. So $G \circ F \in \operatorname{up}(\mathscr{S} g \circ \mathscr{S} f)$ and thus $\mathscr{S}(g \circ f) \sqsupseteq \mathscr{S} g \circ \mathscr{S} f$.

Whenever $H \in \operatorname{up}(\mathscr{S} g \circ \mathscr{S} f)$, we have $H \supseteq G \circ F$ where $F \in \operatorname{up} \mathscr{S} f$, $G \in \operatorname{up} \mathscr{S} g$. Thus $F \supseteq F' \in \operatorname{up} f$, $G \supseteq G' \in \operatorname{up} g$; $H \supseteq G' \circ F' \in \operatorname{up}(g \circ f)$ for some F', G' and so $H \in \operatorname{up}(\mathscr{S}(g \circ f))$. So $\mathscr{S} g \circ \mathscr{S} f \sqsupseteq \mathscr{S}(g \circ f)$.

So $\mathscr{S}(g \circ f) = \mathscr{S} g \circ \mathscr{S} f$.

3°. That it is a prefunctor easily follows from the above.

Suppose f, g are unfixed reloids and $\mathscr{S} f = \mathscr{S} g$. Let $F \in f$, $G \in g$ and thus $\mathscr{S} F = \mathscr{S} G$. It is enough to prove that $F \sim G$.

Really, $\mathscr{S} F = \mathscr{S} G \Rightarrow \mathscr{S} \operatorname{GR} F = \mathscr{S} \operatorname{GR} G \Rightarrow \operatorname{GR} F \sim \operatorname{GR} G \Rightarrow \operatorname{GR} G = (\operatorname{GR} F) \div (\operatorname{dom} G \times \operatorname{im} G) \Leftrightarrow G = F \div (\operatorname{dom} G \times \operatorname{im} G) = \iota_{\operatorname{dom} G, \operatorname{im} G} F$. Similarly $F = \iota_{\operatorname{dom} F, \operatorname{im} F} G$. So $F \sim G$.

□

I yet failed to generalize propositions 2137 and 2138. The generalization may require first research pointfree reloids.

8. More results on restricted identities

In the next three propositions assume $A \in \mathfrak{Z}$, $\mathfrak{A} \ni X \sqsubseteq A$.

PROPOSITION 2148. $\mathrm{id}_X^{\mathbf{Rel}(A)} = \mathrm{id}_{[X]}^{\mathbf{Rel}(A,A)}$.

PROOF. $\mathrm{id}_{[X]}^{\mathbf{Rel}(A,A)} = \mathrm{id}_X^{\mathbf{Rel}(A,A)} = \mathrm{id}_X^{\mathbf{Rel}(A)}$. □

PROPOSITION 2149. $\mathrm{id}_X^{\mathsf{FCD}(A)} = \mathrm{id}_{[X]}^{\mathsf{FCD}(A,A)}$.

PROOF. $\left\langle \mathrm{id}_{[X]}^{\mathsf{FCD}(A,A)} \right\rangle \mathcal{X} = ([\mathcal{X}] \sqcap [X]) \div A = [\mathcal{X} \sqcap X]) \div A = \mathcal{X} \sqcap X = \left\langle \mathrm{id}_X^{\mathsf{FCD}(A)} \right\rangle \mathcal{X}$ for $\mathfrak{A} \ni \mathcal{X} \sqsubseteq A$. □

PROPOSITION 2150. $\mathrm{id}_X^{\mathsf{RLD}(A)} = \mathrm{id}_{[X]}^{\mathsf{RLD}(A,A)}$.

PROOF. $\mathrm{id}_{[X]}^{\mathsf{RLD}(A,A)} = \mathrm{id}_{[X] \div (A \sqcap A)}^{\mathsf{RLD}} \div (A \times A) = \mathrm{id}_X^{\mathsf{RLD}} \div (A \times A) = \mathrm{id}_X^{\mathsf{RLD}(A)}$. □

As a generalization of three last propositions, define for every category $\mathcal{C}$ with restricted identities:

DEFINITION 2151. $\mathrm{id}_X^{\mathcal{C}(A)} = \mathrm{id}_{[X]}^{\mathcal{C}(A,A)}$.

PROPOSITION 2152. $\left\{ \frac{(\mathcal{A} \div A, \mathcal{A} \sqcap A)}{\mathcal{A} \in \mathfrak{F}(U)} \right\}$ is a function and moreover is an order isomorphism for a set $A \subseteq U$.