1°. $(\langle \mathscr{S} f\rangle \mathcal{X}) \div \operatorname{Dst} f = \langle f\rangle \mathcal{X}$. Thus for different f we have different $\mathcal{X} \mapsto \langle \mathscr{S} f\rangle \mathcal{X}$. So it is an injection. That it is a monotone function is obvious.

2°. $\langle \mathscr{S} g \circ \mathscr{S} f\rangle \mathcal{X} = \langle \mathscr{S} g\rangle\langle \mathscr{S} f\rangle \mathcal{X} = \langle \mathscr{S} g\rangle[\langle f\rangle(\mathcal{X} \div \operatorname{Src} f)] = [\langle g\rangle([\langle f\rangle(\mathcal{X} \div \operatorname{Src} f)] \div \operatorname{Src} g)] = [\langle g\rangle(\langle f\rangle(\mathcal{X} \div \operatorname{Src} f) \div \operatorname{Src} g)] = [\langle g\rangle\langle f\rangle(\mathcal{X} \div \operatorname{Src} f)] = [\langle g \circ f\rangle(\mathcal{X} \div \operatorname{Src} f)] = \langle \mathscr{S}(g \circ f)\rangle \mathcal{X}$ for every composable funcoids f and g and an unfixed filter $\mathcal{X}$. Thus $\mathscr{S} g \circ \mathscr{S} f = \mathscr{S}(g \circ f)$.

3°. To prove that it is an order embedding, it is enough to show that $f \not\sim g$ implies $\mathscr{S} f \neq \mathscr{S} g$ (monotonicity is obvious). Let $f \not\sim g$ that is $\iota_{A_0 \sqcup A_1, B_0 \sqcup B_1} f \neq \iota_{A_0 \sqcup A_1, B_0 \sqcup B_1} g$. Then there exist filter $\mathcal{X} \in \mathfrak{F}(A_0 \sqcup A_1)$ such that $\langle \iota_{A_0 \sqcup A_1, B_0 \sqcup B_1} f\rangle \mathcal{X} \neq \langle \iota_{A_0 \sqcup A_1, B_0 \sqcup B_1} g\rangle \mathcal{X}$.

Consequently, $\langle \mathscr{S} f\rangle \mathcal{X} = \langle \mathscr{S} \iota_{A_0 \sqcup A_1, B_0 \sqcup B_1} f\rangle \mathcal{X} \neq \langle \mathscr{S} \iota_{A_0 \sqcup A_1, B_0 \sqcup B_1} g\rangle \mathcal{X} = \langle \mathscr{S} g\rangle \mathcal{X}$.

It remains to prove that $\mathscr{S} G \circ \mathscr{S} F = \mathscr{S}(G \circ F)$ but it is equivalent to $\mathscr{S} g \circ \mathscr{S} f = \mathscr{S}(g \circ f)$ for arbitrarily taken $f \in F$ and $g \in G$, what is already proved above. □

Lemma 2136. For every meet-semilattice $a \not\asymp b$ and $c \sqsupseteq b$ implies $a \sqcap c \not\asymp b$.

Proof. Suppose $a \not\asymp b$. Then there is a non-least x such that $x \sqsubseteq a, b$. Thus $x \sqsubseteq c$, so $x \sqsubseteq a \sqcap c$. We have $a \sqcap c \not\asymp b$. □

FiXme: Since here also for reloids.

Proposition 2137. $\mathscr{S}(X \times Y) = X \times^{\mathsf{pFCD}(\mathfrak{F}(\mho))} Y$ for every unfixed filters X and Y.

Proof. $\mathscr{S}(X \times Y) = \mathscr{S}(X \times_{A,B} Y)$ for arbitrary filters A, and B such that $X \sqsubseteq [A]$ and $Y \sqsubseteq [B]$. So for every unfixed filter $\mathcal{X}$ we have $\langle \mathscr{S}(X \times Y)\rangle \mathcal{X} = \langle \mathscr{S}(X \times_{A,B} Y)\rangle \mathcal{X} = [\langle X \times_{A,B} Y\rangle(\mathcal{X} \div A)] = [\langle (X \div A) \times^{\mathsf{FCD}} (Y \div B)\rangle(\mathcal{X} \div A)]$

Thus if $\mathcal{P} \not\asymp X$ then (by the lemma) $\mathcal{P} \sqcap A \not\asymp X$; $\mathcal{P} \div A \not\asymp X \div A$; $\langle \mathscr{S}(X \times Y)\rangle \mathcal{X} = [Y \div B] = Y$.

if $\mathcal{P} \asymp X$ then $\mathcal{P} \sqcap A \asymp X$; $\mathcal{P} \div A \asymp X \div A$; $\langle \mathscr{S}(X \times Y)\rangle \mathcal{X} = [\bot] = \bot$.

So $\mathscr{S}(X \times Y) = X \times^{\mathsf{pFCD}(\mathfrak{F}(\mho))} Y$. □

Proposition 2138. $\mathscr{S} \operatorname{id}_X = \operatorname{id}_X^{\mathsf{pFCD}(\mathfrak{F}(\mho))}$ for every unfixed filter X.

Proof. For every unfixed filter $\mathcal{X}$ we for arbitrary filters A and B such that $X \sqsubseteq [A] \sqcap [B]$ have $\langle \mathscr{S} \operatorname{id}_X\rangle \mathcal{X} = \left\langle \mathscr{S}[\operatorname{id}_X^{\mathcal{C}(A,B)}]\right\rangle \mathcal{X} = \left\langle \mathscr{S} \operatorname{id}_X^{\mathcal{C}(A,B)}\right\rangle \mathcal{X} = \left[\left\langle \operatorname{id}_X^{\mathcal{C}(A,B)}\right\rangle(\mathcal{X} \div A)\right] = [([\mathcal{X} \div A] \sqcap X) \div B] = [(\mathcal{X} \sqcap X) \div B] = \mathcal{X} \sqcap X$.

Thus $\mathscr{S} \operatorname{id}_X = \operatorname{id}_X^{\mathsf{pFCD}(\mathfrak{F}(\mho))}$. □

7.3. Category RLD.

Definition 2139. $f \div D = (A, B, (\operatorname{GR} f) \div D)$ for every reloid f and a binary relation D.

Category RLD can be considered as a category with restricted identities with $\mathfrak{Z}$ being the set of all small sets, $\mathfrak{A}$ is the set of unfixed filters, projection being the projection function for the equivalence classes of filters, restricted identity being defined by the formula

$$\operatorname{id}_{\mathcal{F}}^{\mathsf{RLD}(A,B)} = \operatorname{id}_{\mathcal{F} \div (A \cap B)}^{\mathsf{RLD}} \div (A \times B).$$

We need to prove that the restricted identities conform to the axioms:

Proof. The first five axioms are obvious. Let's prove the remaining ones:
$\operatorname{id}_{[A]}^{\mathsf{RLD}(A,A)} = \operatorname{id}_{[A] \div A}^{\mathsf{RLD}} \div (A \times A) = \operatorname{id}_A^{\mathsf{RLD}} \div (A \times A) = 1_A^{\mathsf{RLD}}$.