2°. Because of the previous, it is enough to prove that $[f] \in DT \Leftrightarrow f \in Dt$. Really, it is equivalent to $[f] \sqsubseteq T \Leftrightarrow f \sqsubseteq t$ what is obvious.

□

PROPOSITION 2121. If every Hom-set is a distributive lattice, then the poset of unfixed morphisms is a distributive lattice.

PROOF. It follows from the above isomorphism. □

PROPOSITION 2122. If every Hom-set is a co-brouwerian lattice, then the poset of unfixed morphisms is a co-brouwerian lattice.

PROOF. It follows from the above isomorphism and the definition of pseudodifference. □

PROPOSITION 2123. If every Hom-set is a lattice with quasidifference, then the poset of unfixed morphisms is a lattice with quasidifference.

PROOF. It follows from the above isomorphism and the definition of quasidifference. □

PROPOSITION 2124.

1°. If every Hom-set is an atomic lattice, then the poset of unfixed morphisms is an atomic lattice.
2°. If every Hom-set is an atomistic lattice, then the poset of unfixed morphisms is an atomistic lattice.

PROOF. Follows from the above isomorphism. □

6.7. Binary product morphism.

DEFINITION 2125. For a category $\mathcal{C}$ with binary product morphism and $X, Y \in \mathfrak{A}$ define $X \times Y = [X \times_{A,B} Y]$ where $A \in \mathfrak{Z}$, $[A] \sqsupseteq X$, $B \in \mathfrak{Z}$, $[B] \sqsupseteq Y$. (Such A and B exist by an axiom of categories with restricted identities.)

We need to prove validity of this definition:

PROOF. Let $A_0 \in \mathfrak{Z}$, $[A_0] \sqsupseteq X$, $B_0 \in \mathfrak{Z}$, $[B_0] \sqsupseteq Y$, $A_1 \in \mathfrak{Z}$, $[A_1] \sqsupseteq X$, $B_1 \in \mathfrak{Z}$, $[B_1] \sqsupseteq Y$. We need to prove $X \times_{A_0,B_0} Y \sim X \times_{A_1,B_1} Y$, but it trivially follows from an axiom in the definition of category with binary product morphism. □

PROPOSITION 2126. $(X_0 \times Y_0) \sqcap (X_1 \times Y_1) = (X_0 \sqcap X_1) \times (Y_0 \sqcap Y_1)$ for every $X_0, X_1, Y_0, Y_1 \in \mathfrak{A}$.

PROOF. Take $A_0 \in \mathfrak{Z}$, $[A_0] \sqsupseteq X_0$, $B_0 \in \mathfrak{Z}$, $[B_0] \sqsupseteq Y_0$, $A_1 \in \mathfrak{Z}$, $[A_1] \sqsupseteq X_1$, $B_1 \in \mathfrak{Z}$, $[B_1] \sqsupseteq Y_1$.

Then $(X_0 \times Y_0) \sqcap (X_1 \times Y_1) = [X_0 \times_{A_0 \sqcup A_1, B_0 \sqcup B_1} Y_0] \sqcap [X_1 \times_{A_0 \sqcup A_1, B_0 \sqcup B_1} Y_1] = [(X_0 \times_{A_0 \sqcup A_1, B_0 \sqcup B_1} Y_0) \sqcap (X_1 \times_{A_0 \sqcup A_1, B_0 \sqcup B_1} Y_1)] = [(X_0 \sqcap X_1) \times_{A_0 \sqcup A_1, B_0 \sqcup B_1} (Y_0 \sqcap Y_1)] = (X_0 \sqcap X_1) \times (Y_0 \sqcap Y_1)$. □

PROPOSITION 2127. $f \square_{A,B} = f \sqcap (A \times B)$.

PROOF. Take $F \in f$. Let $F' = \iota_{A \sqcup \operatorname{Src} F, B \sqcup \operatorname{Dst} F} F$. We have $F' \in f$. $f \square_{A,B} = [\iota_{A,B} F'] = [\mathcal{E}^{B \sqcup \operatorname{Dst} F, B} \circ F' \circ \mathcal{E}^{A, A \sqcup \operatorname{Src} F}] = [\mathrm{id}^{\mathcal{C}(B \sqcup \operatorname{Dst} F, B)}_{[B]} \circ F' \circ \mathrm{id}^{\mathcal{C}(A, A \sqcup \operatorname{Src} F)}_{[A]}] = [\mathrm{id}^{\mathcal{C}(B \sqcup \operatorname{Dst} F, B)}_{[B]}] \circ [F'] \circ [\mathrm{id}^{\mathcal{C}(A, A \sqcup \operatorname{Src} F)}_{[A]}] = [\mathrm{id}^{\mathcal{C}(B \sqcup \operatorname{Dst} F, B \sqcup \operatorname{Dst} F)}_{[B]}] \circ [F'] \circ [\mathrm{id}^{\mathcal{C}(A \sqcup \operatorname{Src} F, A \sqcup \operatorname{Src} F)}_{[A]}] = [\mathrm{id}^{\mathcal{C}(B \sqcup \operatorname{Dst} F, B \sqcup \operatorname{Dst} F)}_{[B]} \circ F' \circ \mathrm{id}^{\mathcal{C}(A \sqcup \operatorname{Src} F, A \sqcup \operatorname{Src} F)}_{[A]}] = [F' \sqcap (A \times_{A \sqcup \operatorname{Src} F, B \sqcup \operatorname{Dst} F} B)] = [F'] \sqcap [A \times_{A \sqcup \operatorname{Src} F, B \sqcup \operatorname{Dst} F} B] = f \sqcap (A \times B)$. □