The above proposition allows to define:

DEFINITION 2112. $\operatorname{DOM} f = \operatorname{DOM} F$ and $\operatorname{IM} f = \operatorname{IM} F$ for $F \in f$.

6.5. Rectangular restriction.

PROPOSITION 2113. $\iota_{A,B} f = \iota_{A,B} g$ if $f \sim g$.

PROOF. Let $f \sim g$. Then $g = \iota_{\operatorname{Src} g, \operatorname{Dst} g} f$. So $\iota_{A,B} g = \iota_{A,B} \iota_{\operatorname{Src} g, \operatorname{Dst} g} f \sqsubseteq$ (proposition 2090) $\sqsubseteq \iota_{A,B} f$. Similarly, $\iota_{A,B} f \sqsubseteq \iota_{A,B} g$. So $\iota_{A,B} f = \iota_{A,B} g$. □

The above proposition allows to define:

DEFINITION 2114. $\iota_{A,B} F = \iota_{A,B} f$ for an unfixed morphism F and arbitrary $f \in F$.

DEFINITION 2115. $F \square_{A,B} = [\iota_{A,B} F]$ for every unfixed morphism F.

PROPOSITION 2116. $F \square_{A,B} = \operatorname{id}_B \circ F \circ \operatorname{id}_A$ for every unfixed morphism F and objects A and B.

PROOF. Take $f \in F$. $F \square_{A,B} = [\iota_{A,B} F] = [\iota_{A,B} f] = [\mathcal{E}^{\operatorname{Dst} f, B} \circ f \circ \mathcal{E}^{A, \operatorname{Src} f}] = [\operatorname{id}^{\mathcal{C}(\operatorname{Dst} f, B)}_{B \sqcap \operatorname{Dst} f} \circ f \circ \operatorname{id}^{\mathcal{C}(A, \operatorname{Src} f)}_{A \sqcap \operatorname{Src} f}] = [\operatorname{id}^{\mathcal{C}(\operatorname{Dst} f, B)}_B \circ \operatorname{id}^{\mathcal{C}(\operatorname{Dst} f, \operatorname{Dst} f)}_{\operatorname{Dst} f} \circ f \circ \operatorname{id}^{\mathcal{C}(\operatorname{Src} f, \operatorname{Src} f)}_{\operatorname{Src} f} \circ \operatorname{id}^{\mathcal{C}(A, \operatorname{Src} f)}_A] = [\operatorname{id}^{\mathcal{C}(\operatorname{Dst} f, B)}_B \circ f \circ \operatorname{id}^{\mathcal{C}(A, \operatorname{Src} f)}_A] = [\operatorname{id}^{\mathcal{C}(\operatorname{Dst} f, B)}_B] \circ [f] \circ [\operatorname{id}^{\mathcal{C}(A, \operatorname{Src} f)}_A] = \operatorname{id}_B \circ F \circ \operatorname{id}_A$. □

PROPOSITION 2117. $f \square_{A_0, B_0} \square_{A_1, B_1} = f \square_{A_0 \sqcap A_1, A_1 \sqcap B_1}$.

PROOF. From the previous $f \square_{A_0, B_0} \square_{A_1, B_1} = \operatorname{id}_{B_1} \circ \operatorname{id}_{B_0} \circ f \circ \operatorname{id}_{A_0} \circ \operatorname{id}_{A_1} = \operatorname{id}_{B_0 \sqcap B_1} \circ f \circ \operatorname{id}_{A_0 \sqcap A_1} = f \square_{A_0 \sqcap A_1, A_1 \sqcap B_1}$. □

DEFINITION 2118. $f|_X = f \circ \operatorname{id}_X$ for every unfixed morphism f and $X \in \mathfrak{A}$.

OBVIOUS 2119. $(f|_X)|_Y = f_{X \sqcap Y}$.

6.6. Algebraic properties of the lattice of unfixed morphisms. The following proposition allows to easily prove algebraic properties (cf. distributivity) of the poset of unfixed morphisms:

THEOREM 2120. The following are mutually inverse bijections:

1°. Let A and B be objects. $f \mapsto [f]$ and $F \mapsto \iota_{A,B} F$ are mutually inverse order isomorphisms between $\left\{ \frac{f \in \text{unfixed morphisms}}{A \in \operatorname{DOM} f, B \in \operatorname{IM} f} \right\}$ and $\mathcal{C}(A, B)$. If $A = B$ they are also semigroup isomorphisms.

2°. Let T be an unfixed morphism. $f \mapsto [f]$ and $F \mapsto \iota_{\operatorname{Src} t, \operatorname{Dst} t} F$ are mutually inverse order isomorphisms between the lattice DT and Dt whenever $t \in T$.

PROOF. We will prove that these functions are mutually inverse bijections. That they are order-preserving is obvious.

1°. $\iota_{A,B} F \in \mathcal{C}(A, B)$ is obvious.

We need to prove that $[f] \in \left\{ \frac{f \in \text{unfixed morphisms}}{A \in \operatorname{DOM} f, B \in \operatorname{IM} f} \right\}$. For this it's enough to prove $A \in \operatorname{DOM}[f] \wedge B \in \operatorname{IM}[f]$ what is the same as $A \in \operatorname{DOM} f \wedge B \in \operatorname{IM} f$ what follows from proposition 2071.

Because $f \mapsto [f]$ is an injection, it is enough[1] to prove that $\iota_{A,B}[f] = f$. Really, $\iota_{A,B}[f] = \iota_{A,B} f = f$.

That they are semigroup isomorphisms follows from the already proved formula $[g \circ f] = [g] \circ [f]$.

[1] https://math.stackexchange.com/a/3007051/4876