1°. If every Hom-set is a join-semilattice, then the poset of unfixed morphism is a join-semilattice.
2°. If every Hom-set is a join-semilattice, then the poset of unfixed morphism is a meet-semilattice.

PROOF. Let f and g be arbitrary morphisms.

$$[f] \sqcup [g] = [\iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} f] \sqcup [\iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} g] = \text{(obvious 2106)} = [\iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} f \sqcup \iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} g]$$

and

$$[f] \sqcap [g] = [\iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} f] \sqcap [\iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} g] = \text{(obvious 2106)} = [\iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} f \sqcap \iota_{\operatorname{Src} f \sqcup \operatorname{Src} g, \operatorname{Dst} f \sqcup \operatorname{Dst} g} g].$$

□

COROLLARY 2109. If every Hom-set is a lattice, then the poset of unfixed morphisms is a lattice.

THEOREM 2110. Meet of nonempty set of unfixed morphisms exists provided that the orders of Hom-sets are posets, every nonempty subset of which has a meet, and our category is with ordered domain and image and that morphisms $\mathcal{E}$ are metamonovalued and metainjective.

PROOF. Let S be a nonempty set of unfixed morphisms. Take an arbitrary unfixed morphism $f \in S$. Take an arbitrary $F \in f$. Let $A = \operatorname{Src} F$ and $B = \operatorname{Dst} F$. $\bigsqcap S = \bigsqcap \langle f \sqcap \rangle^* S = \bigsqcap \langle [F] \sqcap \rangle^* S = \bigsqcap \left\{ \frac{[F] \sqcap [G]}{g \in S, G \in g} \right\} = \bigsqcap \left\{ \frac{[\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F \sqcap \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} G]}{g \in S, G \in g} \right\}$

We will prove $\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F \sqcap \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} G \sim F \sqcap \iota_{A,B} G$.

$\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F \sqcap \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} G \sqsubseteq \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F$ and $\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} \iota_{A,B} \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F = \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F$, thus by being with ordered domain and image

$$\begin{aligned}
&\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F \sqcap \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} G = \\
&\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} \iota_{A,B} (\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F \sqcap \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} G) = \\
&\text{(by being metamonovalued and metainjective)} = \\
&\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} (\iota_{A,B} \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} F \sqcap \iota_{A,B} \iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} G) = \\
&\iota_{A \sqcup \operatorname{Src} G, B \sqcup \operatorname{Dst} G} (\iota_{A,B} F \sqcap \iota_{A,B} G) \sim \iota_{A,B} F \sqcap \iota_{A,B} G = F \sqcap \iota_{A,B} G.
\end{aligned}$$

Due the proved equivalence we have $\bigsqcap S = \bigsqcap \left\{ \frac{[F \sqcap \iota_{A,B} G]}{g \in S, G \in g} \right\}$. Now we can apply proposition 2107: $\bigsqcap S = \left[\bigsqcap \left\{ \frac{F \sqcap \iota_{A,B} G}{g \in S, G \in g} \right\} \right]$. We have provided an explicit formula for $\bigsqcap S$. □

The poset of unfixed morphisms may be not a complete lattice even if every Hom-set is a complete lattice. We will show this below for funcoids.

6.4. Domain and image of unfixed morphisms.

PROPOSITION 2111. $\operatorname{IM} f = \left\{ \frac{Y \in \mathfrak{Z}}{\mathrm{id}_Y \circ [f] = [f]} \right\}$; $\operatorname{DOM} f = \left\{ \frac{X \in \mathfrak{Z}}{[f] \circ \mathrm{id}_X = [f]} \right\}$.

PROOF. We will prove only the first, as the second is similar. $\mathrm{id}_Y \circ [f] = [f] \Leftrightarrow \mathrm{id}_Y^{\mathcal{C}(Y \sqcup \operatorname{Dst} f, Y \sqcup \operatorname{Dst} f)} \circ \mathcal{E}^{\operatorname{Dst} f, Y \sqcup \operatorname{Dst} f} \circ f = \mathcal{E}^{\operatorname{Dst} f, Y \sqcup \operatorname{Dst} f} \circ f \Leftrightarrow \mathrm{id}_{[Y] \sqcap [\operatorname{Dst} f]}^{\mathcal{C}(\operatorname{Dst} f, Y \sqcup \operatorname{Dst} f)} \circ f = \mathcal{E}^{\operatorname{Dst} f, Y \sqcup \operatorname{Dst} f} \circ f \Leftrightarrow \mathcal{E}^{Y \sqcup \operatorname{Dst} f, \operatorname{Dst} f} \circ \mathrm{id}_{[Y] \sqcap [\operatorname{Dst} f]}^{\mathcal{C}(\operatorname{Dst} f, Y \sqcup \operatorname{Dst} f)} \circ f = f \Leftrightarrow \mathrm{id}_{[Y] \sqcap [\operatorname{Dst} f]}^{\mathcal{C}(\operatorname{Dst} f, \operatorname{Dst} f)} \circ f = f Y \in \operatorname{IM} f$. □