OBVIOUS 2062. $\iota_{B_0,B_1} f \sqsubseteq f$.

PROPOSITION 2063. $\iota_{\mathrm{Src}\, f,\mathrm{Dst}\, f} f = f$.

PROOF. $\iota_{\mathrm{Src}\, f,\mathrm{Dst}\, f} f = \mathcal{E}_{\mathcal{C}}^{\mathrm{Dst}\, f,\mathrm{Dst}\, f} \circ f \circ \mathcal{E}_{\mathcal{C}}^{\mathrm{Src}\, f,\mathrm{Src}\, f} = 1_{\mathcal{C}}^{\mathrm{Dst}\, f} \circ f \circ 1_{\mathcal{C}}^{\mathrm{Src}\, f} = f$. □

PROPOSITION 2064. The function $\iota_{B_0,B_1}|_{f\in\mathrm{Hom}_{\mathcal{C}}(A_0,A_1)}$ is injective, provided that $A_0 \sqsubseteq B_0$ and $A_1 \sqsubseteq B_1$.

PROOF. Because $\mathcal{E}_{\mathcal{C}}^{A_1,B_1}$ is a monomorphism and $\mathcal{E}_{\mathcal{C}}^{A_0,B_0}$ is an epimorphism. □

COROLLARY 2065. The function $\iota_{B_0,B_1}|_{f\in\mathrm{Hom}_{\mathcal{C}}(A_0,A_1)}$ is order embedding if $A_0 \sqsubseteq B_0 \wedge A_1 \sqsubseteq B_1$ for ordered categories with restricted identities.

3. Image and domain

Let define that $\mathscr{S}\mathcal{A} = \left\{ \frac{K\in\mathfrak{Z}}{\exists X\in\mathcal{A}:X\subseteq K} \right\}$ holds not only for filters but for any set $\mathcal{A}$ of sets.

OBVIOUS 2066. $\mathscr{S}\mathcal{A} \supseteq \mathcal{A}$.

DEFINITION 2067.
1°. $\mathrm{IM}\, f = \left\{ \frac{Y\in\mathfrak{Z}}{\mathcal{E}_{\mathcal{C}}^{Y,\mathrm{Dst}\, f}\circ\mathcal{E}_{\mathcal{C}}^{\mathrm{Dst}\, f,Y}\circ f=f} \right\} = \left\{ \frac{Y\in\mathfrak{Z}}{\mathrm{id}^{\mathcal{C}(\mathrm{Dst}\, f,\mathrm{Dst}\, f)}_{[Y]\sqcap[\mathrm{Dst}\, f]}\circ f=f} \right\}$;
2°. $\mathrm{DOM}\, f = \left\{ \frac{X\in\mathfrak{Z}}{f\circ\mathcal{E}_{\mathcal{C}}^{\mathrm{Src}\, f,X}\circ\mathcal{E}_{\mathcal{C}}^{X,\mathrm{Src}\, f}=f} \right\} = \left\{ \frac{X\in\mathfrak{Z}}{f\circ\mathrm{id}^{\mathcal{C}(\mathrm{Src}\, f,\mathrm{Src}\, f)}_{[X]\sqcap[\mathrm{Src}\, f]}=f} \right\}$.

DEFINITION 2068.
1°. $\mathrm{Im}\, f = \left\{ \frac{Y\in\mathrm{IM}\, f}{Y\sqsubseteq\mathrm{Dst}\, f} \right\}$;
2°. $\mathrm{Dom}\, f = \left\{ \frac{X\in\mathrm{DOM}\, f}{X\sqsubseteq\mathrm{Src}\, f} \right\}$.

PROPOSITION 2069.
1°. $\mathrm{IM}\, f = \mathscr{S}\,\mathrm{Im}\, f$;
2°. $\mathrm{DOM}\, f = \mathscr{S}\,\mathrm{Dom}\, f$;
3°. $\mathrm{Im}\, f = \langle \mathrm{Dst}\, f\cap\rangle^*\,\mathrm{IM}\, f$;
4°. $\mathrm{Dom}\, f = \langle \mathrm{Dst}\, f\cap\rangle^*\,\mathrm{DOM}\, f$.

PROOF. $\mathrm{IM}\, f = \left\{ \frac{Y\in\mathfrak{Z}}{\mathrm{id}^{\mathcal{C}(\mathrm{Dst}\, f,\mathrm{Dst}\, f)}_{[Y]\sqcap[\mathrm{Dst}\, f]}\circ f=f} \right\}$.
Suppose $Y \in \mathrm{IM}\, f$. Then take $Y' = Y \sqcap \mathrm{Dst}\, f$. We have $Y \sqsupseteq Y'$ and $Y' \in \mathrm{Im}\, f$. So $Y \in \mathscr{S}\,\mathrm{Im}\, f$. If $Y \in \mathscr{S}\,\mathrm{Im}\, f$ then $Y \in \mathrm{IM}\, f$ obviously. So $\mathrm{IM}\, f = \mathscr{S}\,\mathrm{Im}\, f$.
$\langle \mathrm{Dst}\, f\cap\rangle^*\,\mathrm{IM}\, f \subseteq \mathrm{Im}\, f$ is obvious. If $\mathrm{Im}\, f \subseteq \langle \mathrm{Dst}\, f\cap\rangle^*\,\mathrm{IM}\, f$ is also obvious.
The rest follows from symmetry. □

CONJECTURE 2070. $\mathrm{Im}\, f$ may be not a filter for an injective category with restricted morphisms.

PROPOSITION 2071. $\mathrm{Dst}\, f \in \mathrm{Im}\, f$; $\mathrm{Src}\, f \in \mathrm{Dom}\, f$ for every morphism f of a category with restricted identities.

PROOF. Prove $\mathrm{Dst}\, f \in \mathrm{Im}\, f$ (the other is similar): We need to prove that $\mathcal{E}_{\mathcal{C}}^{\mathrm{Dst}\, f,\mathrm{Dst}\, f}\circ\mathcal{E}_{\mathcal{C}}^{\mathrm{Dst}\, f,\mathrm{Dst}\, f}\circ f = f$ what follows from $\mathcal{E}_{\mathcal{C}}^{\mathrm{Dst}\, f,\mathrm{Dst}\, f}\circ\mathcal{E}_{\mathcal{C}}^{\mathrm{Dst}\, f,\mathrm{Dst}\, f} = 1^{\mathrm{Dst}\, f}$. □

PROPOSITION 2072. $\mathrm{IM}\, f$, $\mathrm{Im}\, f$, $\mathrm{DOM}\, f$, $\mathrm{Dom}\, f$ are upper sets.

PROOF. For $\mathrm{Im}\, f$, $\mathrm{Dom}\, f$ it follows from the previous proposition.
For $\mathrm{IM}\, f$, $\mathrm{DOM}\, f$ it follows from the thesis for $\mathrm{Im}\, f$, $\mathrm{Dom}\, f$. □

DEFINITION 2073.