

Convergence of funcoids*

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Abstract

Considered convergence for funcoids (a generalization of proximity spaces). This article is a part of my Algebraic General Topology research.

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1 Draft status

It is a draft.

2 Common

See [1] for the definition of funcoid. We will use the reversed order of filters as in [1].

3 Convergence

Definition 1. A filter \mathcal{F} converges to a filter \mathcal{A} regarding a funcoid μ ($\mathcal{F} \xrightarrow{\mu} \mathcal{A}$) iff $\mathcal{F} \subseteq \langle \mu \rangle \mathcal{A}$.¹

Definition 2. A funcoid f converges to a filter \mathcal{A} regarding a funcoid μ ($f \xrightarrow{\mu} \mathcal{A}$) iff $\text{im } f \subseteq \langle \mu \rangle \mathcal{A}$ that is iff $\text{im } f \xrightarrow{\mu} \mathcal{A}$.

Definition 3. A funcoid f converges to a filter \mathcal{A} on a filter \mathcal{B} regarding a funcoid μ iff $f|_{\mathcal{B}} \xrightarrow{\mu} \mathcal{A}$.

Remark 4. We can define also convergence for a reloid $f: f \xrightarrow{\mu} \mathcal{A} \Leftrightarrow \text{im } f \subseteq \langle \mu \rangle \mathcal{A}$ or what is the same $f \xrightarrow{\mu} \mathcal{A} \Leftrightarrow (\text{FCD}) f \xrightarrow{\mu} \mathcal{A}$.

Theorem 5. Let f, g, μ, ν are funcoids, \mathcal{A} is a filter. If $f \xrightarrow{\mu} \mathcal{A}$,

$$g|_{\langle \mu \rangle \mathcal{A}} \in C(\mu \cap (\langle \mu \rangle \mathcal{A})^2; \nu)$$

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1. This generalizes the standard definition of filter convergent to a point or to a set.

and $\langle \mu \rangle \mathcal{A} \supseteq \mathcal{A}$ then $g \circ f \xrightarrow{\nu} \langle g \rangle \mathcal{A}$.

Proof. $\text{im } f \subseteq \langle \mu \rangle \mathcal{A}$; $\langle g \rangle \text{im } f \subseteq \langle g \rangle \langle \mu \rangle \mathcal{A}$; $\text{im}(g \circ f) \subseteq \langle g|_{\langle \mu \rangle \mathcal{A}} \rangle \langle \mu \rangle \mathcal{A}$; $\text{im}(g \circ f) \subseteq \langle g|_{\langle \mu \rangle \mathcal{A}} \rangle \langle \mu \cap (\langle \mu \rangle \mathcal{A})^2 \rangle \mathcal{A}$; $\text{im}(g \circ f) \subseteq \langle g|_{\langle \mu \rangle \mathcal{A}} \circ (\mu \cap (\langle \mu \rangle \mathcal{A})^2) \rangle \mathcal{A}$; $\text{im}(g \circ f) \subseteq \langle \nu \circ g|_{\langle \mu \rangle \mathcal{A}} \rangle \mathcal{A}$; $\text{im}(g \circ f) \subseteq \langle \nu \circ g \rangle \mathcal{A}$; $\text{im}(g \circ f) \subseteq \langle \nu \rangle \langle g \rangle \mathcal{A}$; $g \circ f \xrightarrow{\nu} \langle g \rangle \mathcal{A}$. \square

Corollary 6. Let f, g, μ, ν are funcoids, \mathcal{A} is a filter. If $f \xrightarrow{\mu} \mathcal{A}$, $g \in C(\mu; \nu)$ and $\langle \mu \rangle \mathcal{A} \supseteq \mathcal{A}$ then $g \circ f \xrightarrow{\nu} \langle g \rangle \mathcal{A}$.

Proof. From the last theorem and a theorem in [2]. \square

The following is the theorem about convergence of a continuous funcoid:

Theorem 7. If $f \in C(\mu; \nu)$ then $f|_{\langle \mu \rangle \mathcal{A}} \xrightarrow{\nu} \langle f \rangle \mathcal{A}$ (for any funcoids μ and n and a filter \mathcal{A}).

Proof. $f|_{\langle \mu \rangle \mathcal{A}} \xrightarrow{\nu} \langle f \rangle \mathcal{A} \Leftrightarrow \text{im } f|_{\langle \mu \rangle \mathcal{A}} \subseteq \langle \nu \rangle \langle f \rangle \mathcal{A} \Leftrightarrow \langle f \rangle \langle \mu \rangle \mathcal{A} \subseteq \langle \nu \rangle \langle f \rangle \mathcal{A} \Leftrightarrow \langle f \circ \mu \rangle \mathcal{A} \subseteq \langle \nu \circ f \rangle \mathcal{A} \Leftrightarrow f \circ \mu \subseteq \nu \circ f \Leftrightarrow f \in C(\mu; \nu)$. \square

4 Limit

Definition 8. $\lim^\mu f = a$ iff $f \xrightarrow{\mu} \{a\}$ for a T_2 -separable funcoid μ and a non-empty funcoid f .

It is defined correctly, that is f has no more than one limit.

Proof. Let $\lim^\mu f = a$ and $\lim^\mu f = b$. Then $\text{im } f \subseteq \langle \mu \rangle \{a\}$ and $\text{im } f \subseteq \langle \mu \rangle \{b\}$

Because $f \neq \emptyset$ we have $\text{im } f \neq \emptyset$; $\langle \mu \rangle \{a\} \cap \langle \mu \rangle \{b\} \neq \emptyset$; $\{b\} \cap \langle \mu^{-1} \rangle \langle \mu \rangle \{a\} \neq \emptyset$; $\{b\} \cap \langle \mu^{-1} \circ \mu \rangle \{a\} \neq \emptyset$; $\{a\}[\mu^{-1} \circ \mu] \{b\}$. Because μ is T_2 -separable we have $a = b$. \square

Definition 9. $\lim_{\mathcal{B}}^\mu f = \lim^\mu (f|_{\mathcal{B}})$.

Remark 10. We can also in an obvious way define limit of a reloid.

Bibliography

- [1] Victor Porton. Funcoids and reloids. At <http://www.mathematics21.org/binaries/funcoids-reloids.pdf>.
- [2] Victor Porton. Generalized continuousness. At <http://www.mathematics21.org/binaries/continuousness.pdf>.