Hyperfuncoids

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This is a rough partial draft.

1 Hyperfuncoids

Let $\mathfrak{A}$ is an indexed family of sets.

Products are $\prod A$ for $A \in \prod \mathfrak{A}$.

Problem 1. Is $\prod^{\mathsf{FCD}}$ a bijection from hyperfuncoids $\mathfrak{F}\Gamma$ to:

1. prestaroids on $\mathfrak{A}$;
2. staroids on $\mathfrak{A}$;
3. completary staroids on $\mathfrak{A}$?

If yes, is up^{Γ} defining the inverse bijection?

If not, characterize the image of the function $\prod^{\mathsf{FCD}}$ defined on $\mathfrak{F}\Gamma$.

Alternatively (differently for the infinite dimensional case!) define Γ as the set of intersections of *sets with holes* that is $\prod \mathfrak{A} \setminus \prod A$ where $A \sqsubseteq \mathfrak{A}$. In other words, it is the set Γ^* of complements of elements of the set Γ.

Theorem 2. For every anchored relation f on powersets, $f = \prod^{\mathrm{Anch}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f$.

Proof. We need to prove only $f \sqsubseteq \prod^{\mathrm{Anch}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f$.

Fix $n \in \mathrm{arity}\, \mathfrak{A}$. Let $A \in \prod_{(\mathrm{arity}\, f) \setminus \{n\}} \mathfrak{A}_i$.

[TODO: Define the complement.]

Define $g(A) = \left(\prod_{(\mathrm{arity}\, f) \setminus \{n\}} A_i \times \langle f \rangle_n A\right) \cup \left(\overline{\prod_{(\mathrm{arity}\, f) \setminus \{n\}} A_i} \times 1\right)$ for $A \in \prod_{(\mathrm{arity}\, f) \setminus \{n\}} \mathfrak{A}_i$. Obviously $g(A) \in \Gamma^*$.

Let $X \in \prod_{(\mathrm{arity}\, f) \setminus \{n\}} \mathfrak{A}_i$.

If $0 \neq X \sqsubseteq A$ then $\langle g(A) \rangle_n X = \langle f \rangle_n A \sqsupseteq \langle f \rangle_n X$.

If $X \not\sqsubseteq A$ then $\langle g(A) \rangle_n X = 1$.

So $\langle G(A) \rangle_n \sqsupseteq \langle f \rangle_n$ and thus $G(A) \sqsupseteq f$.

For a given f, we have $\langle g(A) \rangle_n A = \langle f \rangle_n A$. Thus for every $A \in \prod_{(\mathrm{arity}\, f) \setminus \{n\}} \mathfrak{A}_i$ we have $\langle f \rangle_n A \sqsubseteq \left\langle \prod^{\mathsf{cStrd}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f \right\rangle_n A$ and so $f \sqsubseteq \prod^{\mathrm{Anch}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f$. □

Corollary 3.

1. If f is a prestaroid, $f = \prod^{\mathsf{pStrd}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f$.
2. If f is a staroid, $f = \prod^{\mathsf{Strd}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f$.
3. If f is a completary staroid, $f = \prod^{\mathsf{cStrd}(\mathfrak{A})} \mathrm{up}^{\Gamma^*} f$.