1 Hyperfuncoids

Let $A$ is an indexed family of sets. Products are $Q_A$ for $A \in \prod A$.

**Problem 1.** Is $\prod^\operatorname{FCD}$ a bijection from hyperfuncoids $\exists \Gamma$ to:
1. prestaroids on $A$;
2. staroids on $A$;
3. completary staroids on $A$?

If yes, is $\up^\Gamma$ defining the inverse bijection? If not, characterize the image of the function $\prod^\operatorname{FCD}$ defined on $\exists \Gamma$.

Alternatively (differently for the infinite dimensional case!) define $\Gamma$ as the set of intersections of sets with holes that is $\prod A \setminus \prod A$ where $A \subseteq A$. In other words, it is the set $\Gamma^*$ of complements of elements of the set $\Gamma$.

**Theorem 2.** For every anchored relation $f$ on powersets, $f = \prod^\operatorname{Anch}(A) \up^\Gamma^* f$.

**Proof.** We need to prove only $f \subseteq \prod^\operatorname{Anch}(A) \up^\Gamma^* f$.

Fix $n \in \operatorname{arity} A$. Let $A \in \prod_{(\operatorname{arity} f) \setminus \{n\}} A_i$.

[TODO: Define the complement.]

Define $g(A) = \left( \prod_{(\operatorname{arity} f) \setminus \{n\}} A_i \times (f)_n A \right) \cup \left( \prod_{(\operatorname{arity} f) \setminus \{n\}} A_i \times 1 \right)$ for $A \in \prod_{(\operatorname{arity} f) \setminus \{n\}} A_i$.

Obviously $g(A) \in \Gamma^*$.

Let $X \in \prod_{(\operatorname{arity} f) \setminus \{n\}} A_i$.

If $0 \neq X \subseteq A$ then $(g(A))_n X = (f)_n A \supseteq (f)_n X$.

If $X \not\subseteq A$ then $(g(A))_n X = 1$.

So $(g(A))_n \supseteq (f)_n$ and thus $g(A) \supseteq f$.

For a given $f$, we have $(g(A))_n A = (f)_n A$. Thus for every $A \in \prod_{(\operatorname{arity} f) \setminus \{n\}} A_i$ we have $(f)_n A \subseteq \left( \prod^\operatorname{Strd}(A) \up^\Gamma^* f \right)_n A$ and so $f \subseteq \prod^\operatorname{Anch}(A) \up^\Gamma^* f$. $\square$

**Corollary 3.**

1. If $f$ is a prestaroid, $f = \prod^\operatorname{pStrd}(A) \up^\Gamma^* f$.
2. If $f$ is a staroid, $f = \prod^\operatorname{Strd}(A) \up^\Gamma^* f$.
3. If $f$ is a completary staroid, $f = \prod^\operatorname{cStrd}(A) \up^\Gamma^* f$. 