Limit of discontinuous functions

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Abstract

Defined (generalized) limit for any (not necessarily continuous) functions under ceratin conditions. Particularly limit of any function from any topological vector space to any topological space is defined.

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Draft status

This text is only an idea, not a complete article.

The definition

Let μ and ν are funcoids [1], G is a group of functions.

Let D is a set such that $\forall r \in G \colon \operatorname{im} r \subseteq D \wedge \forall x, y \in D \exists r \in G \colon r(x) = y$.

We require that μ and any $r \in G$ commute, that is $\mu \circ r = r \circ \mu$.

We are going to considere (generalized) limits of functions acting from μ to ν. (The functions in consideration are not required to be continuous.)

Remark 1. Most typically G is the group of translations of some topological vector space and μ is the topology of that space.

Definition 2. $\lim f \overset{\text{def}}{=} \{\nu \circ f \circ r \mid r \in G\}$ for any funcoid f.

Definition 3. $\lim_x f \overset{\text{def}}{=} \lim f|_{\langle\mu\rangle\{x\}}$.

Obvious 4. $\lim_x f = \left\{\nu \circ f|_{\langle\mu\rangle\{x\}} \circ r \mid r \in G\right\}$.

We want to identify values y in $\operatorname{dom} \nu$ with certain generalized limits. I will denote τ the mapping between these.

Definition 5. $\tau(y) \overset{\text{def}}{=} \{\nu \circ (\langle\mu\rangle\{x\} \times \{y\}) \mid x \in D\}$.

Proposition 6. $\tau(y) \overset{\text{def}}{=} \{\nu \circ (\langle\mu\rangle\{x\} \times \{y\}) \circ r \mid r \in G\}$ for any $x \in D$.

Proof. $(\langle\mu\rangle\{x\} \times y) \circ r = \langle r\rangle\langle\mu\rangle\{x\} \times \{y\} = \langle\mu\rangle\langle r\rangle\{x\} \times y = \langle\mu\rangle\{r x\} \times \{y\} = \langle\mu\rangle\{x'\} \times \{y\}$ where $x' \in D$. So

$$\nu \circ (\langle\mu\rangle\{x\} \times \{y\}) \circ r = \nu \circ (\langle\mu\rangle\{x'\} \times \{y\}).$$

Reversely $\langle\mu\rangle\{x\}\times\{y\}=(\langle\mu\rangle\{x\}\times\{y\})\circ e$ where e is the identify element of G. So

$$\nu\circ(\langle\mu\rangle\{x\}\times\{y\})=\nu\circ(\langle\mu\rangle\{x\}\times\{y\})\circ e. \qquad \square$$

Corollary 7. $\tau(y)=\lim(\langle\mu\rangle\{x\}\times\{y\})$ for any $x\in D$.

Corollary 8. $\tau(y)=\lim_x y$ for any $x\in D$.

We should prove that τ is a bijection in the case of certain T_x-separability properties of ν.

Yet to do

We yet need to prove miscellaneous properties of generalized limit. Especially we should prove that (provided certain T_x-separability properties of ν) that in the case of non-singularity the definition of generalized limit coincides well with the customary limit. We also should introduce n-ary functions extended on values which generalized limits take, so that we could be able for example to add two generalized limits.

We should study differential equations generalized for the derivative of non-smooth functions.

Bibliography

[1] Victor Porton. Funcoids and reloids. At http://www.mathematics21.org/binaries/funcoids-reloids.pdf.