# Circuitoids, a Generalization of Categories

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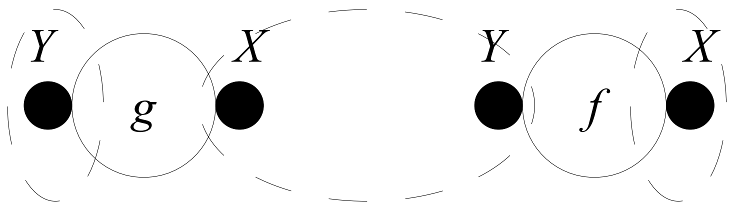
**The attempt presented in this article to generalize several directions in my research was to cumbersome. It appears that it's better to consider every case individually without this generalization. This article is a blind valley in research. I don't suggest to read the below.**

This article is not to be published, because the author is not an expert in category theory. But it may be used as a part of my future writings on the topic of general topology.

## 1 Circuitoids

Consider the following Venn diagram:




On this diagram the set (of the cardinality four) of arguments (denoted as $X$ and $Y$) of binary relations $f$ and $g$ (denoted as circles) is split into the collection $G$ of three sets (let's call them *edges*[1]) denoted by the dashed lines.

I will call a *proposed solution* assigning every edge a value. I will call a *solution* a proposed solution such that its values are related by the binary relations whose arguments belong to the edges.

The set of values assigned to the argument $X$ of $f$ and the argument $Y$ of $g$ such that there exists a solution containing these values is essentially the composition $g \circ f$ of binary relations $f$ and $g$.

Thus we have expressed a composition of binary relations in the form of Venn diagrams.

Now let move on to a definition of composition of arbitrary (having an arbitrary, possibly infinite, number of arguments) relations in term of Venn diagrams.

**Remark 1.** Let $F$ is a family of relations. Imagine that every relation $F_i$ is an electronic component with $\operatorname{card} \operatorname{dom} F_i$ contacts. The partition $G$ specifies which contacts are connected together. The set $Z$ denotes a set of "external" contacts which we measure. This results in a new relation (I call it *graph-composition*) of $\operatorname{card} Z$ arguments. See a formal definition of this (*the circuitoid of relations*) below.

We will first do it in a more abstract way, using "morphisms" defined below (instead of relations) having an arbitrary number of arguments.

Now we will formalize something similar to a category but each morphism having an arbitrary (possibly infinite) number of arguments, instead of two arguments of a morphism of a category.

**Definition 2.** *An argumentoid is:*

1. *a small set* Hom *(the set of morphisms);*
2. *a small set* $\operatorname{Arg} f$ *(set of arguments) for every* $f \in \operatorname{Hom}$*;*

1. The term *edge* is from hypergraph theory.

3. *a small function* $\mathrm{Type} = \mathrm{Type}(f; x)$ *defined for every* $f \in \mathrm{Hom}$ *and* $x \in \mathrm{Arg}\, f$.

**Remark 3.** Two morphisms may be composed on arguments only when these arguments have the same type. So type is like an object (source or destination of a morphism in category theory). See below for an exact formulation.

Let $F$ is a family of morphisms of an argumentoid. Then the corresponding *set of vertices*

$$V(F) = \bigcup \{\{i\} \times \mathrm{Arg}\, F_i \mid i \in \mathrm{dom}\, F\}.$$

**Obvious 4.** *Vertices are pairs* $(i; x)$ *where* $i \in \mathrm{dom}\, F$ *and* $x \in \mathrm{Arg}\, F_i$.

**Definition 5.** *Having an argumentoid, a composition situation is:*

1. *a family of morphisms* $F$;
2. *a partition* $G$ *(set of edges) of the set* $V(F)$, *such that* $\mathrm{Type}(F_i; x) = \mathrm{Type}(F_{i'}; x')$ *for every vertices* $(i; x)$ *and* $(i'; x')$ *lie in the same part;*
3. *a set* $Z \in \mathscr{P}V$ *(external set).*

**Definition 6.** *Having a vertex* $v$ *I will denote* $[v]$ *the edge which contains it.*

**Definition 7.** *We will denote* $T(g) = \mathrm{Type}(F_i; x)$ *where* $(i; x) \in g$ *for every* $g \in G$.

**Remark 8.** We may somehow attempt to define associativity of graph-composition. But this is not urgently required for my research, so I now skip this topic without investigating.

**Definition 9.** *The type of a graph composition is* $\lambda v \in Z: T([v])$.

**Definition 10.** *A circuitoid is an argumentoid equipped with a function (graph-composition) which assigns a morphism to every composition situation, such that the type of graph composition is as specified above.*

We may attempt to define associativity for circuitoids and thus form a structure similar to a precategory.

# 2 Circuitoid of relations

**Definition 11.** *The argumentoid of relations is the argumentoid whose morphisms are pairs* $(A; f)$ *where* $A$ *is a small family of sets and* $f$ *is a relation on the set* $\prod A$, *and* $\mathrm{Arg}(A; f) = A$, $\mathrm{Type}((A; f); x) = A_x$.

I will also denote $\mathrm{Val}(A; f) = f$.

**Definition 12.** *Having a composition situation for the argumentoid of relations, we define a proposed solution as a function* $s \in \prod_{g \in G} T(g)$.

**Definition 13.** *A proposed solution is a solution iff for every* $i \in \mathrm{dom}\, F$ *we have*

$$(\lambda x \in \mathrm{Arg}\, F_i: s_{[(i;x)]}) \in \mathrm{Val}\, F_i.$$

The *graph-composition* for relations is the type of graph composition together with $Z$-ary relation:

$$\left\{L \in \prod Z \mid \exists s: (s \text{ is a solution} \wedge \forall v \in Z: L_v = s_{[v]})\right\}.$$

*The circuitoid of relations* is the argumentoid of relations together with this graph-composition.

**Theorem 14.** *The circuitoid of relations is really a circuitoid.*

**Proof.** We need to prove that

$$\left\{ L \in \prod Z \mid \exists s \colon (s \text{ is a solution} \wedge s|_Z = L) \right\}$$

is a relation on the set $\prod \lambda v \in Z \colon T([v])$ that is on the set $\prod_{v \in Z} T([v])$. It follows from the fact that $L \in \prod_{v \in Z} T([v])$ for every $L \in \prod Z$ that is $L_v \in T([v])$ that is $s_{[v]} \in T([v])$ what follows from the definition of proposed solution. □

# 3 Other examples of circuitoids

Other examples of circuitoids are *circuitoid of funcoids* and *circuitoid of reloids*.

Some of these are present in the following draft (which is incomplete as of now):

http://www.mathematics21.org/binaries/nary.pdf