

Open Problems in Algebraic General Topology*

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Abstract

This document lists in one place all conjectures and open problems in my Algebraic General Topology research which were yet not solved. This document also contains other relevant materials such as proved theorems related with the conjectures.

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Organizational info

Discuss these problems and their solutions in the `algebraic-general-topology` Google group.

See <http://www.mathematics21.org/algebraic-general-topology.html> for more details.

See also <http://filters.wikidot.com/open-problems> for open problems about filters.

Read <http://www.mathematics21.org/solvers.html> if you solved any of the below problems in order that I could nominate you for Abel Prize if I found your solutions worth it.

Misc

Conjecture 1. *If $\mathcal{A} \in \mathfrak{F}$, B is a set then $\lambda \mathcal{Y} \in \mathfrak{F}(B): \mathcal{A} \times^{\text{RLD}} \mathcal{Y}$ is a complete lattice homomorphism of the lattice \mathfrak{F} to a full sublattice the lattice $\text{RLD}(\text{Base}(\mathcal{A}); B)$, if also $\mathcal{A} \neq 0^{\mathfrak{F}(\text{Base}(\mathcal{A}))}$ then it is an isomorphism.*

Conjecture 2. *A reloid f is monovalued iff*

$$\forall g \in \text{RLD}(\text{Src } f; \text{Dst } f): (g \subseteq f \Rightarrow \exists \mathcal{A} \in \mathfrak{F}(\text{Src } f): g = f|_{\mathcal{A}}).$$

*. This document has been written using the GNU $\text{T}_{\text{E}}\text{X}_{\text{M}}\text{A}_{\text{C}}\text{S}$ text editor (see www.texmacs.org).

Conjecture 3. *If f and g are composable reloids, then*

$$g \circ f = \bigcup \{G \circ F \mid F \in \text{atoms } f, G \in \text{atoms } g\}.$$

Conjecture 4. *$g \circ f = \bigcap \{\uparrow^{\text{FCD}(\text{Src } f; \text{Dst } g)}(G \circ F) \mid F \in \text{up } f, G \in \text{up } g\}$ for every composable funcoids f and g .*

Conjecture 5. *The filtrator of funcoids is:*

1. *with separable core;*
2. *with co-separable core.*

Conjecture 6. *Let A, B, C be sets. There exist a set $R \in \mathcal{P}\text{FCD}(A; B)$ of funcoids and a funcoid $f \in \text{FCD}(B; C)$ such that*

$$f \circ \bigcup R \neq \bigcup \langle f \circ \rangle R.$$

Complete funcoids and reloids

Question 7. *Is $\text{ComplFCD}(A; B)$ a co-brouwerian lattice?*

Conjecture 8. *Composition of complete reloids is complete.*

Conjecture 9. *$\text{Compl } f \cap \text{Compl } g = \text{Compl}(f \cap g)$ for every reloids $f, g \in \text{RLD}(A; B)$ (for every sets A, B).*

Conjecture 10. *$\text{Compl } f = f \setminus^* (\Omega(\text{Src } f) \times^{\text{FCD}} 1^{\mathfrak{F}(\text{Dst } f)})$ for every funcoid f .*

Conjecture 11. *$\text{Compl } f = f \setminus^* (\Omega(\text{Src } f) \times^{\text{RLD}} 1^{\mathfrak{F}(\text{Dst } f)})$ for every reloid f .*

Question 12. *Is $\text{ComplRLD}(A; B)$ a distributive lattice? Is $\text{ComplRLD}(A; B)$ a co-brouwerian lattice? (for every sets A and B).*

Conjecture 13. *Let A, B, C be sets. If $f \in \text{FCD}(B; C)$ is a complete funcoid and $R \in \mathcal{P}\text{FCD}(A; B)$ then $f \circ \bigcup R = \bigcup \langle f \circ \rangle R$.*

Conjecture 14. *Let A, B, C be sets. If $f \in \text{RLD}(B; C)$ is a complete reloid and $R \in \mathcal{P}\text{RLD}(A; B)$ then $f \circ \bigcup R = \bigcup \langle f \circ \rangle R$.*

Two above conjectures can be weakened:

Conjecture 15. *Let A, B, C be sets. If $f \in \text{FCD}(B; C)$ is a discrete funcoid and $R \in \mathcal{P}\text{FCD}(A; B)$ then $f \circ \bigcup R = \bigcup \langle f \circ \rangle R$.*

Conjecture 16. *Let A, B, C be sets. If $f \in \text{RLD}(B; C)$ is a discrete reloid and $R \in \mathcal{P}\text{RLD}(A; B)$ then $f \circ \bigcup R = \bigcup \langle f \circ \rangle R$.*

Relationships of functors and rewrites

Conjecture 17. $(\text{RLD})_{\text{in}}(g \circ f) = (\text{RLD})_{\text{in}}g \circ (\text{RLD})_{\text{in}}f$ for every composable functors f and g .

Conjecture 18. $(\text{RLD})_{\text{out}}I_{\mathcal{A}}^{\text{FCD}} = I_{\mathcal{A}}^{\text{RLD}}$ for every f.o. \mathcal{A} .

Conjecture 19. $(\text{RLD})_{\text{in}}$ is not a lower adjoint (in general).

Conjecture 20. $(\text{RLD})_{\text{out}}$ is neither a lower adjoint nor an upper adjoint (in general).

Conjecture 21. $\text{dom}(\text{RLD})_{\text{in}}f = \text{dom } f$ and $\text{im}(\text{RLD})_{\text{in}}f = \text{im } f$ for every functor f .

Conjecture 22. If $\mathcal{A} \times^{\text{RLD}} \mathcal{B} \subseteq (\text{RLD})_{\text{in}}f$ then $\mathcal{A} \times^{\text{FCD}} \mathcal{B} \subseteq f$ for every functor f and $\mathcal{A} \in \mathfrak{F}(\text{Src } f)$, $\mathcal{B} \in \mathfrak{F}(\text{Dst } f)$.

Connectedness of functors and rewrites

Conjecture 23. A filter \mathcal{A} is connected regarding a functor μ iff \mathcal{A} is connected for every binary relation $F \in \text{up } \mu$ (considered as a functor).

Conjecture 24. A filter \mathcal{A} is connected regarding a rewrite f iff it is connected regarding the functor $(\text{FCD})f$.

Conjecture 25. Let \mathcal{A} is an f.o. and F is a binary relation on $A \times B$ for some sets A, B .

\mathcal{A} is connected regarding $\uparrow^{\text{FCD}(A;B)}F$ iff \mathcal{A} is connected regarding $\uparrow^{\text{RLD}(A;B)}F$.

Algebraic properties of S and S^*

Conjecture 26. $S(S(f)) = S(f)$ for

1. any endo-rewrite f ;
2. any endo-functor f .

Conjecture 27. For any endo-rewrite f

1. $S(f) \circ S(f) = S(f)$;
2. $S^*(f) \circ S^*(f) = S^*(f)$;
3. $S(f) \circ S^*(f) = S^*(f) \circ S(f) = S^*(f)$.

Conjecture 28. $S(f) \circ S(f) = S(f)$ for any endo-functor f .

Oblique products

Conjecture 29. $\mathcal{A} \times_F^{\text{RLD}} \mathcal{B} \subset \mathcal{A} \times \mathcal{B}$ for some f.o. \mathcal{A}, \mathcal{B} .

A stronger conjecture:

Conjecture 30. $\mathcal{A} \times_F^{\text{RLD}} \mathcal{B} \subset \mathcal{A} \times \mathcal{B} \subset \mathcal{A} \times^{\text{RLD}} \mathcal{B}$ for some f.o. \mathcal{A}, \mathcal{B} . Particularly, is this formula true for $\mathcal{A} = \mathcal{B} = \Delta \cap \uparrow^{\text{R}}(0; +\infty)$?

Conjecture 31. $(\mathcal{A} \times \mathcal{B}) \cup (\mathcal{A} \times \mathcal{B}) = \mathcal{A} \times^{\text{RLD}} \mathcal{B}$ for every f.o. \mathcal{A}, \mathcal{B} .

Compactness and Heine-Cantor theorem

$\forall \mathcal{F} \in \mathfrak{F}: (\mathcal{F} \cap \text{im } f \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} [f] \mathcal{F})$ or equivalently

$$\forall \mathcal{F} \in \mathfrak{F}: (\langle f^{-1} \rangle \mathcal{F} \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} \subseteq \langle f^{-1} \rangle \mathcal{F})$$

is a possible definition of *compact* funcoïd. (A special case of this definition was hinted by VICTOR PETROV.) How this is related with open covers and finite covers from the traditional definition of compactness? Does compactness imply completeness?

Generalize Heine-Cantor theorem for funcoïds and reloïds.