

Open Problems in Algebraic General Topology*

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February 6, 2009

Abstract

This document lists in one place all conjectures and open problems in my Algebraic General Topology research which were yet not solved. This document also contains other relevant materials such as proved theorems related with the conjectures.

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Organizational info

Discuss these problems and their solutions in the `algebraic-general-topology` Google group.

See <http://www.mathematics21.org/algebraic-general-topology.html> for more details.

Read <http://www.mathematics21.org/solvers.html> if you solved any of the below problems in order that I could nominate you for Abel Prize.

Misc

Conjecture 1. If $\mathcal{A} \in \mathcal{F}$ then $\mathcal{A} \times^{\text{RLD}}$ is a complete lattice homomorphism of the lattice \mathcal{F} to a full sublattice the lattice RLD , if also $\mathcal{A} \neq \emptyset$ then it is an isomorphism.

Conjecture 2. If f, g are full functors (generalized closures) then $f \cap^{\text{FCD}} g$ is a full functor (generalized closure).

Conjecture 3. If f, g are binary relations then $f \cap^{\text{FCD}} g$ is a binary relation.

The conjecture 3 easily follows from the conjecture 2. I'm almost sure that these two important conjectures are true.

Conjecture 4. If a retractor is monovalued then it is a monovalued function restricted to some filter.

Conjecture 5. Composition of two atomic retractors is atomic or empty.

Conjecture 6. Atomic retractors are monovalued.

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Conjecture 7. A reloid f is monovalued iff $\forall g \in \text{RLD}: (g \subseteq f \Rightarrow \exists \mathcal{A} \in \mathcal{F}: g = f|_{\mathcal{A}})$.

Conjecture 8. If f, g, h are reloids then

1. $f \circ (g \cup h) = f \circ g \cup f \circ h$;
2. $(g \cup h) \circ f = g \circ f \cup h \circ f$.

Conjecture 9. A monovalued reloid restricted to an atomic filter is atomic or empty.

Conjecture 10. A (monovalued) function restricted to an atomic filter is atomic or empty.

Relationships of funcoids and reloids

Conjecture 11. $(\text{FCD})(\text{RLD})_{\text{in}}f = f$ for any funcoid f .

Conjecture 12. For any funcoids f and g

1. $(\text{RLD})_{\text{out}}(g \circ f) = (\text{RLD})_{\text{out}}g \circ (\text{RLD})_{\text{out}}f$;
2. $(\text{RLD})_{\text{in}}(g \circ f) = (\text{RLD})_{\text{in}}g \circ (\text{RLD})_{\text{in}}f$.

Definition 13. I will call a reloid *convex* iff it is a union of direct products.

Proposition 14. A reloid is convex iff it is a union of direct products of atomic filters.

Proof. From the theorem which tells that every direct product (in the sense of the theory of reloids) of filters is an (infinite) union of direct products (in the sense of the theory of reloids) of atomic filters. \square

Conjecture 15. For a convex reloid f

1. $(\text{RLD})_{\text{out}}(\text{FCD})f = f$;
2. $(\text{RLD})_{\text{in}}(\text{FCD})f = f$.

Conjecture 16. $\bigcup^{\text{FCD}} \langle (\text{FCD}) \rangle S = (\text{FCD}) \bigcup^{\mathcal{F}} S$ if S is a set of reloids.

Conjecture 17. For any funcoid f and reloid g

$$(\text{RLD})_{\text{out}}f \subseteq g \subseteq (\text{RLD})_{\text{in}}f \Leftrightarrow (\text{FCD})g = f.$$

Algebraic properties of S and S^*

Conjecture 18. $S(S(f)) = S(f)$ for

1. any reloid f ;
2. any funcoid f .

Conjecture 19. For any reloid f

1. $S(f) \circ S(f) = S(f)$;
2. $S^*(f) \circ S^*(f) = S^*(f)$;
3. $S(f) \circ S^*(f) = S^*(f) \circ S(f) = S^*(f)$.

Conjecture 20. $S(f) \circ S(f) = S(f)$ for any funcoid f .

Compactness and Heine-Cantor theorem

$\forall \mathcal{F} \in \mathcal{F}: (\mathcal{F} \cap \text{im } f \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} [f] \mathcal{F})$ or equivalently

$$\forall \mathcal{F} \in \mathcal{F}: (\langle f^{-1} \rangle \mathcal{F} \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} \subseteq \langle f^{-1} \rangle \mathcal{F})$$

is a possible definition of *compact* funcoid. (A special case of this definition was hinted by VICTOR PETROV.) How this is related with open covers and finite covers from the traditional definition of compactness? Does compactness imply fullness?

Generalize Heine-Cantor theorem for funcoids and reloids.