

Open Problems in Algebraic General Topology*

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Abstract

This document lists in one place all conjectures and open problems in my Algebraic General Topology research which were yet not solved. This document also contains other relevant materials such as proved theorems related with the conjectures.

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Organizational info

Discuss these problems and their solutions in the algebraic-general-topology Google group.

See <http://www.mathematics21.org/algebraic-general-topology.html> for more details.

See also <http://filters.wikidot.com/open-problems> for open problems about filters.

Read <http://www.mathematics21.org/solvers.html> if you solved any of the below problems in order that I could nominate you for Abel Prize if I would worth find your solutions worth it.

Misc

Conjecture 1. *If $\mathcal{A} \in \mathfrak{F}$ then $\mathcal{A} \times^{\text{RLD}}$ is a complete lattice homomorphism of the lattice \mathfrak{F} to a full sublattice the lattice RLD, if also $\mathcal{A} \neq \emptyset$ then it is an isomorphism.*

Conjecture 2. *If a reloid is monovalued then it is a monovalued function restricted to some filter.*

Conjecture 3. *A reloid f is monovalued iff $\forall g \in \text{RLD}: (g \subseteq f \Rightarrow \exists \mathcal{A} \in \mathfrak{F}: g = f|_{\mathcal{A}})$.*

Conjecture 4. *If f, g, h are reloids then*

1. $f \circ (g \cup h) = f \circ g \cup f \circ h;$

*. This document has been written using the GNU $\text{T}_{\text{E}}\text{X}_{\text{M}}\text{A}^{\text{C}}\text{S}$ text editor (see www.texmacs.org).

$$2. (g \cup h) \circ f = g \circ f \cup h \circ f.$$

Conjecture 5. *A monovalued reloid restricted to an atomic filter is atomic or empty.*

Conjecture 6. *A (monovalued) function restricted to an atomic filter is atomic or empty.*

Conjecture 7. *The filtrator of funcoids is:*

1. *with separable core;*
2. *with co-separable core.*

Conjecture 8. *The set of discrete funcoids is the center of the lattice of funcoids.*

Conjecture 9. *There exist a set R of funcoids and a funcoid f such that*

$$f \circ \bigcup^{\text{FCD}} R \neq \bigcup^{\text{FCD}} \langle f \circ \rangle R.$$

Complete funcoids and reloids

Question 10. *Is ComplFCD a co-brouwerian lattice?*

Conjecture 11. *If a reloid is both complete and co-complete then it is discrete.*

Conjecture 12. *Composition of complete reloids is complete.*

Conjecture 13. $\text{Compl } f \cap^{\text{RLD}} \text{Compl } g = \text{Compl}(f \cap^{\text{RLD}} g)$ for every reloids f and g .

Conjecture 14. $\text{Compl}(\bigcup^{\text{RLD}} R) = \bigcup^{\text{RLD}} \langle \text{Compl} \rangle R$ for every set R of reloids.

Conjecture 15. $\text{Compl } \text{CoCompl } f = \text{CoCompl } \text{Compl } f = \text{Cor } f$ for every reloid f .

Question 16. *Is ComplRLD a distributive lattice? Is ComplRLD a co-brouwerian lattice?*

Conjecture 17. *If f is a complete funcoid and R is a set of funcoids then $f \circ \bigcup^{\text{FCD}} R = \bigcup^{\text{FCD}} \langle f \circ \rangle R$.*

Conjecture 18. *If f is a complete reloid and R is a set of reloids then $f \circ \bigcup^{\text{RLD}} R = \bigcup^{\text{RLD}} \langle f \circ \rangle R$.*

Two above conjectures can be weakened:

Conjecture 19. *If f is a discrete funcoid and R is a set of funcoids then $f \circ \bigcup^{\text{FCD}} R = \bigcup^{\text{FCD}} \langle f \circ \rangle R$.*

Conjecture 20. *If f is a discrete reloid and R is a set of reloids then $f \circ \bigcup^{\text{RLD}} R = \bigcup^{\text{RLD}} \langle f \circ \rangle R$.*

Relationships of functors and relops

Conjecture 21. For any functors f and g

1. $(\text{RLD})_{\text{out}}(g \circ f) = (\text{RLD})_{\text{out}}g \circ (\text{RLD})_{\text{out}}f$;
2. $(\text{RLD})_{\text{in}}(g \circ f) = (\text{RLD})_{\text{in}}g \circ (\text{RLD})_{\text{in}}f$.

Definition 22. I will call a reloid convex iff it is a union of direct products.

Proposition 23. A reloid is convex iff it is a union of direct products of atomic filters.

Proof. From the theorem which tells that every direct product (in the sense of the theory of relops) of filters is an (infinite) union of direct products (in the sense of the theory of relops) of atomic filters. \square

Conjecture 24. For a convex reloid f

1. $(\text{RLD})_{\text{out}}(\text{FCD})f = f$;
2. $(\text{RLD})_{\text{in}}(\text{FCD})f = f$.

Connectedness of functors and relops

Conjecture 25. A filter \mathcal{A} is connected regarding a functor μ iff \mathcal{A} is connected for every binary relation $F \in \text{up } \mu$ (considered as a functor).

Conjecture 26. A filter \mathcal{A} is connected regarding a reloid f iff it is connected regarding the functor $(\text{FCD})f$.

Conjecture 27. A filter is connected regarding a binary relation considered as a functor iff it is connected regarding this binary relation considered as a reloid.

Algebraic properties of S and S^*

Conjecture 28. $S(S(f)) = S(f)$ for

1. any reloid f ;
2. any functor f .

Conjecture 29. For any reloid f

1. $S(f) \circ S(f) = S(f)$;
2. $S^*(f) \circ S^*(f) = S^*(f)$;
3. $S(f) \circ S^*(f) = S^*(f) \circ S(f) = S^*(f)$.

Conjecture 30. $S(f) \circ S(f) = S(f)$ for any functor f .

Compactness and Heine-Cantor theorem

$\forall \mathcal{F} \in \mathfrak{F}: (\mathcal{F} \cap \text{im } f \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} [f] \mathcal{F})$ or equivalently

$$\forall \mathcal{F} \in \mathfrak{F}: (\langle f^{-1} \rangle \mathcal{F} \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} \subseteq \langle f^{-1} \rangle \mathcal{F})$$

is a possible definition of *compact* funcoïd. (A special case of this definition was hinted by VICTOR PETROV.) How this is related with open covers and finite covers from the traditional definition of compactness? Does compactness imply completeness?

Generalize Heine-Cantor theorem for funcoïds and reloïds.