

# Open Problems in Algebraic General Topology\*

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## Abstract

This document lists in one place all conjectures and open problems in my Algebraic General Topology research which were yet not solved. This document also contains other relevant materials such as proved theorems related with the conjectures.

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## Organizational info

Discuss these problems and their solutions in the algebraic-general-topology Google group.

See <http://www.mathematics21.org/algebraic-general-topology.html> for more details.

See also <http://filters.wikidot.com/open-problems> for open problems about filters.

Read <http://www.mathematics21.org/solvers.html> if you solved any of the below problems in order that I could nominate you for Abel Prize if I would worth find your solutions worth it.

## Misc

**Conjecture 1.** *If  $\mathcal{A} \in \mathfrak{F}$  then  $\mathcal{A} \times^{\text{RLD}}$  is a complete lattice homomorphism of the lattice  $\mathfrak{F}$  to a full sublattice the lattice RLD, if also  $\mathcal{A} \neq \emptyset$  then it is an isomorphism.*

**Conjecture 2.** *If  $f, g$  are full functors (generalized closures) then  $f \cap^{\text{FCD}} g$  is a full functor (generalized closure).*

**Conjecture 3.** *If  $f, g$  are binary relations then  $f \cap^{\text{FCD}} g$  is a binary relation.*

The conjecture 3 easily follows from the conjecture 2. I'm almost sure that these two important conjectures are true.

**Conjecture 4.** *If a reloid is monovalued then it is a monovalued function restricted to some filter.*

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**Conjecture 5.** A reloid  $f$  is monovalued iff  $\forall g \in \text{RLD}: (g \subseteq f \Rightarrow \exists \mathcal{A} \in \mathfrak{F}: g = f|_{\mathcal{A}})$ .

**Conjecture 6.** If  $f, g, h$  are reloids then

1.  $f \circ (g \cup h) = f \circ g \cup f \circ h$ ;
2.  $(g \cup h) \circ f = g \circ f \cup h \circ f$ .

**Conjecture 7.** A monovalued reloid restricted to an atomic filter is atomic or empty.

**Conjecture 8.** A (monovalued) function restricted to an atomic filter is atomic or empty.

**Conjecture 9.** The filtrator of funcoids is:

1. with separable core;
2. with co-separable core.

**Conjecture 10.** The set of discrete funcoids is the center of the lattice of funcoids.

## Relationships of funcoids and reloids

**Conjecture 11.**  $(\text{FCD})(\text{RLD})_{\text{in}}f = f$  for any funcoid  $f$ .

**Conjecture 12.** For any funcoids  $f$  and  $g$

1.  $(\text{RLD})_{\text{out}}(g \circ f) = (\text{RLD})_{\text{out}}g \circ (\text{RLD})_{\text{out}}f$ ;
2.  $(\text{RLD})_{\text{in}}(g \circ f) = (\text{RLD})_{\text{in}}g \circ (\text{RLD})_{\text{in}}f$ .

**Definition 13.** I will call a reloid convex iff it is a union of direct products.

**Proposition 14.** A reloid is convex iff it is a union of direct products of atomic filters.

**Proof.** From the theorem which tells that every direct product (in the sense of the theory of reloids) of filters is an (infinite) union of direct products (in the sense of the theory of reloids) of atomic filters.  $\square$

**Conjecture 15.** For a convex reloid  $f$

1.  $(\text{RLD})_{\text{out}}(\text{FCD})f = f$ ;
2.  $(\text{RLD})_{\text{in}}(\text{FCD})f = f$ .

**Conjecture 16.**  $\bigcup^{\text{FCD}} \langle (\text{FCD}) \rangle S = (\text{FCD}) \bigcup^{\mathfrak{F}} S$  if  $S$  is a set of reloids.

**Conjecture 17.** For any funcoid  $f$  and reloid  $g$

$$(\text{RLD})_{\text{out}}f \subseteq g \subseteq (\text{RLD})_{\text{in}}f \Leftrightarrow (\text{FCD})g = f.$$

## Algebraic properties of $S$ and $S^*$

**Conjecture 18.**  $S(S(f)) = S(f)$  for

1. any reloid  $f$ ;
2. any funcooid  $f$ .

**Conjecture 19.** For any reloid  $f$

1.  $S(f) \circ S(f) = S(f)$ ;
2.  $S^*(f) \circ S^*(f) = S^*(f)$ ;
3.  $S(f) \circ S^*(f) = S^*(f) \circ S(f) = S^*(f)$ .

**Conjecture 20.**  $S(f) \circ S(f) = S(f)$  for any funcooid  $f$ .

## Compactness and Heine-Cantor theorem

$\forall \mathcal{F} \in \mathfrak{F}: (\mathcal{F} \cap \text{im } f \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\}[f]\mathcal{F})$  or equivalently

$$\forall \mathcal{F} \in \mathfrak{F}: (\langle f^{-1} \rangle \mathcal{F} \neq \emptyset \Rightarrow \exists \alpha: \{\alpha\} \subseteq \langle f^{-1} \rangle \mathcal{F})$$

is a possible definition of *compact* funcooid. (A special case of this definition was hinted by VICTOR PETROV.) How this is related with open covers and finite covers from the traditional definition of compactness? Does compactness imply fullness?

Generalize Heine-Cantor theorem for funcooids and reloids.